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Abstract

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ON THE DEVELOPMENT OF ELECTROSTATIC INSTABILITY OF A PLASMA IN A STRONG MAGNETIC FIELD

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1. It is known that the state of a plasma often becomes unstable even under a small deviation from thermodynamic equilibrium ([1]). As a result of the development of perturbations in such an unstable plasma, it passes into a turbulent state. If the plasma turbulence is weak (i.e., if the energy of the growing collective oscillations remains significantly less than the energy of the thermal motion of the plasma particles), then the equations of the quasilinear theory of plasma ([2]) can be used to study the process of development of perturbations in an unstable plasma and the properties of the resulting weakly turbulent state.

In practice, however, one encounters considerable difficulties in solving the quasilinear equations; an essential simplification of the problem arises in the case when the spectrum of nonequilibrium noises in the turbulent plasma is one-dimensional. The one-dimensionality of the spectrum is, generally speaking, connected with the presence of a distinguished direction in the system; we wish to draw attention to the circumstance that, during the development of instability in a plasma located in a strong magnetic field (which selects a direction), the spectrum of plasma oscillations is almost one-dimensional.

2. Let us consider the problem of the development of electrostatic instability in a fully ionized rarefied plasma, assuming that the constant magnetic field in which the plasma is located is so large that the ratios of the gas pressure nT to the magnetic pressure $H^2/8\pi$ and of the frequency of the developing oscillations ω to the gyrofrequency are small*. In this case the electric fields of the oscillations are potential, and perturbations of the magnetic field are absent.

The system of equations of quasilinear kinetics can be obtained under these conditions by expanding the exact quasilinear equations in $1/\omega_H$ and has the form

$$\frac{\partial f^\alpha}{\partial t} = \frac{\partial}{\partial v}\left(D_\parallel^\alpha \frac{\partial f^\alpha}{\partial v}\right), \tag{1}$$

$$D_{\parallel}^{\alpha} = \int dk\, B^{\alpha}(\mathbf{k})\, \varepsilon(\mathbf{k})\, \delta(\omega_k - k_{\parallel} v), \tag{2}$$

$$\frac{\partial \varepsilon(\mathbf{k})}{\partial t} = \sum_{\alpha} A^{\alpha}(\mathbf{k})\, \varepsilon(\mathbf{k}) \int \frac{\partial f^{\alpha}}{\partial v}\, \delta(\omega_k - k_{\parallel} v)\, dv. \tag{3}$$

Here $f^{\alpha}(v) = \int F^{\alpha}(\mathbf{v})\, d\mathbf{v}_{\perp}$; $F^{\alpha}(\mathbf{v})$ is the velocity distribution function of particles of species α; v is the projection of the velocity on the direction of the magnetic field; $v_{\perp}$ is the transverse component of the velocity; $\varepsilon(\mathbf{k}) = E_k^2/8\pi$ is the spectral density of the electrostatic energy of the oscillations; ω_k is the frequency of the wave $\mathbf{k}$;

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* That is, $\omega \ll \Omega_H = eH/Mc$ for ionic and $\omega \ll \omega_H = eH/mc$ for electronic Langmuir oscillations.

the form of the functions A and B depends on which waves (ion or electron Langmuir oscillations) are excited in the unstable plasma*.

Equation (1) describes the diffusion of particles in velocity space under the action of plasma oscillations; the diffusion coefficient $D_{\parallel}$ is proportional to the energy of the plasma waves whose phase velocity $\omega_k/k_{\parallel}$ is equal to the velocity of the so-called "resonant particles" (equation (2)); the change in the spectrum of the oscillations is determined by the interaction of the given wave with the resonant particles (equation (3)).

3. If at the initial instant the right-hand side of (3) is positive (i.e., there is a "beam" in the plasma), then the waves grow; it is essential that "oblique" waves ($k_{\perp} \neq 0$) then grow more slowly than purely longitudinal ones—the derivative $\{\partial A_k/\partial k_{\perp}^2\}_{k_{\perp}=0} \equiv -A_1 < 0$. Therefore the spectrum of oscillations developing in the plasma becomes anisotropic, and this anisotropy increases with time. If the initial noise is sufficiently small, then before appreciable diffusion in velocity space begins the spectrum will already become practically one-dimensional. Indeed, integrating (3) with respect to time, we obtain**

$$\varepsilon_k(t) = \varepsilon_k^0 \exp \int_0^t \left(A_0 - A_1 k_{\perp}^2 + ...\right) \frac{\partial f}{\partial v}\, dt \tag{4}$$

$$\left(A_0 \equiv \{A\}_{k_{\perp}=0}\right).$$

The integral

$$\int_0^t \frac{\partial f}{\partial v}\, dt$$

increases monotonically with time; therefore, for sufficiently large t, the higher terms of the expansion of the exponent in (4) become insignificant, and the energy distribution over transverse wave numbers is Gaussian,

$$\varepsilon_k(t) = C \exp - (k_\perp/\varkappa)^2, \qquad C = \varepsilon_k^0 \exp \int_0^t A_0 \frac{\partial f}{\partial v}\, dt \tag{4'}$$

with width

$$\varkappa(t) = \left(\int_0^t A_1 \frac{\partial f}{\partial v}\, dt \right)^{-1/2} . \tag{5}$$

Using expressions (2) and (4), we obtain the value of the particle diffusion coefficient in velocity space as a result of interaction with the waves:

$$D_\parallel = \pi \frac{B_0}{|v_r - v|} C \varkappa^2, \qquad B_0 \equiv \{B\}_{k_\perp = 0}, \qquad v_r = \left\{ \frac{d\omega_k}{dk_\parallel} \right\}_{k_\perp = 0} . \tag{6}$$

The change of the diffusion coefficient with time is determined by the expression

$$\frac{\partial D_\parallel}{\partial t} = D_\parallel A_0 \frac{\partial f}{\partial v} \left\{ 1 - \frac{1}{\int A_0 \frac{\partial f}{\partial v}\, dt} + ... \right\}, \tag{7}$$

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* For example, for ion-acoustic oscillations,

$$A_k = \pi \omega_k \frac{T_e}{m} \left(1 + k^2 R_D^2 + 4k_\perp^2 T_e / m\Omega_H^2\right)^{-3/2}, \qquad \Omega_H = \frac{eH}{Mc},$$

R_D is the Debye radius, T_e is the electron temperature.

** For simplicity, in what follows we consider the excitation of ion-acoustic waves in a plasma with "hot" electrons and "cold" ions; we neglect ion diffusion in velocity space and its contribution to the growth rate of plasma waves.

i.e., with time the integral $\left|\int_0^t A_0 \frac{\partial f}{\partial v}\, dt\right| \sim \ln \frac{\varepsilon}{\varepsilon^0}$ grows, and the law of variation of the diffusion coefficient (7) approaches the law of variation for a one-dimensional spectrum. With the aid of formulas (5) and (6) one can relate the half-width of the spectrum $\chi(t)$ to the diffusion coefficient $D_\parallel$ or to the intensity of the plasma noise ε,

$$\chi(t) = \left(\int_0^t A_1 \frac{\partial f}{\partial v}\, dt\right)^{-1/2} = \left(\frac{A_1}{A_0} \ln \frac{D_\parallel}{D_\parallel^2}\right)^{-1/2} = \left(\frac{A_1}{A_0} \ln \frac{\varepsilon}{\varepsilon^0}\right)^{-1/2}. \quad (8)$$

4. Thus, the equations describing the development of the instability asymptotically approach the equations for a one-dimensional spectrum:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v} D_\parallel \frac{\partial f}{\partial v}; \quad (1a)$$

$$\frac{\partial D_\parallel}{\partial t} = A_0 D_\parallel \frac{\partial f}{\partial v}. \quad (3a)$$

Solving these equations, one can find the noise intensity $D_\parallel(t)$ and the width of the energy distribution of the oscillations over transverse wave numbers (equation (8)).

As is known, the one-dimensional equations (1a)–(3a) have an integral that makes it possible to find the noise spectrum or the diffusion coefficient at any instant of time from the change in the distribution function. Indeed, from (1a) and (3a) it follows that

$$\frac{\partial D_\parallel}{\partial t} = D_\parallel A_0 \frac{\partial f}{\partial v} = A_0 \frac{\partial}{\partial t} \int f\, dv,$$

so that

$$D_\parallel^\infty(v) = A_0 \int_{v_1}^{v} \left(f^\infty(v') - f^0(v')\right) dv, \quad (9)$$

where f^0, f^∞ are the velocity distribution functions of the resonant particles at the initial and final instants of time, and $D_\parallel^\infty$ is the diffusion coefficient in the final state (the initial noises may here be neglected, $D_\parallel^0 = 0$). The initial distribution function f^0 is known; the distribution function in the final state must have the form of a "plateau" : $f^\infty = \text{const}$, with the value f^∞ and the positions of the boundaries of the "plateau" $v_{1,2}$ determined by the equalities [1,2]

$$f^\infty = \int_{v_1}^{v_2} \frac{f^0(v)\, dv}{(v_2 - v_1)}, \qquad f^\infty = f^0(v_1) = f^0(v_2).$$

5. Thus, relations (8) and (9) completely determine the spectrum of plasma oscillations arising as a result of the development of the instability (relaxation of the "beam") in a strong magnetic field. The initial (thermal) noise level is very small because of the small volume of the region

of wave-number space $\mathbf{k}$ in which non-damped waves are possible; therefore, as the instability develops, the noise spectrum must be practically one-dimensional.

In nearly longitudinal oscillations, the component of the electric field transverse to the magnetic field is very small. In this case the "anomalous" diffusion of particles across the magnetic field H in a weakly inhomogeneous plasma, caused by the drift of particles in random electric fields of low-frequency oscillations, is also small; the magnitude of the coefficient of "anomalous" diffusion is

$$D_\perp = \frac{\langle(\Delta x_\perp)^2\rangle}{\Delta t} = \int dk\, \frac{k_\perp^2}{k^2}\frac{c^2|E_k|^2}{H^2}\delta(\omega_k - k_\parallel v)$$

can be found by using expression (41): for example, for an electron having velocity v,

$$D_\perp = \frac{1}{2\pi}\frac{\varkappa^2}{k_\parallel^2}\frac{D_\parallel(v)}{\omega_H^2},$$

where $k_\parallel$ is the root of the equation $\omega(k_\parallel) = k_\parallel v$.

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CITED LITERATURE

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2. A. A. Vedenov, E. P. Velikhov, R. Z. Sagdeev, Conf. on Plasma Physics and Controlled Thermonuclear Fusion, Salzburg, 1961.

Note: Figure translations are in progress. See original paper for figures.