A. PAVLIKOVSKI

1962

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Abstract

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A. PAVLIKOVSKI

ON THE SEPARATION OF THE SELF-CONSISTENT FIELD IN THE SUPERFLUID MODEL OF THE NUCLEUS

(Presented by Academician N. N. Bogolyubov, March 3, 1962)

Let us consider a system of N fermions described by the Hamiltonian

$$H = \sum_i \varepsilon_i a_i^+ a_i + \sum_{ijkl} V_{ijkl} a_i^+ a_j^+ a_k a_l, \tag{1}$$

where a_i^+, a_i are the creation and annihilation operators of fermions in the single-particle state $|i\rangle$. In the second term of the Hamiltonian (1) there is, generally speaking, a certain self-consistent field. This field is equal to

$$\sum_{il} K_{il} a_i^+ a_i, \tag{2}$$

where

$$K_{il} = \sum_{jk\sigma} \vartheta_{ijkl} \psi_\sigma^*(j) \psi_\sigma(k); \tag{3}$$

$$\vartheta_{ijkl} = V_{ijkl} - V_{jikl} - V_{ijlk} + V_{jilk}; \tag{4}$$

the functions $\psi_\alpha(i) = \langle i|\alpha\rangle$ are solutions of the Hartree–Fock equations

$$\varepsilon_i \psi_\alpha(i) + \sum_l K_{il} \psi_\alpha(l) = E_\alpha \psi_\alpha(i); \tag{5}$$

the index σ enumerates the states in which the particles are found. The quantities E_σ are the particle energies in the self-consistent field.

Let us write the Hamiltonian (1) in the form

$$H = H_1 + H_2,$$

$$H_1 = \sum_i \varepsilon_i a_i^+ a_i + \sum_{il} K_{il} a_i^+ a_l + C, \tag{6}$$

$$H_2 = \sum_{ijkl} V_{ijkl} a_i^+ a_j^+ a_k a_l - \sum_{il} K_{il} a_i^+ a_l - C,$$

where

$$C = -\sum_{\substack{ijkl \\ \sigma,\sigma'}} (V_{ijkl} - V_{ijlk}) \psi_\sigma^*(i) \psi_{\sigma'}^*(j) \psi_{\sigma'}(k) \psi_\sigma(l). \tag{7}$$

After diagonalization the operator H_1 takes the form

$$H_1 = \sum_\alpha E_\alpha a_\alpha^+ a_\alpha + C, \tag{8}$$

where a_α^+, a_α are the creation and annihilation operators of fermions in the single-particle state $|\alpha\rangle$. In the Hamiltonian (6) the self-consistent field has been explicitly separated.

Let us apply the procedure given above to the Hamiltonian of a system of N nucleons with a pairing interaction

$$H = \sum_s \varepsilon_s \left(a_{s+}^+ a_{s+} + a_{s-}^+ a_{s-}\right) - G \sum_{ss'} a_{s+}^+ a_{s-}^+ a_{s'-} a_{s'+}, \tag{9}$$

where $a_{s\rho}^+, a_{s\rho}$ $(\rho = \pm)$ are the creation and annihilation operators of nucleons in the one-particle state $|s\rho\rangle$, $G > 0$. The solution of the Hartree-Fock equations in this case is to some extent trivial. The states $|\alpha\rangle$ are simply equal to the initial states $|s\rho\rangle$. For the one-particle energies E_s we obtain

$$E_s = \begin{cases} \varepsilon_s - G & \text{for levels } s \text{ on which there are two nucleons,} \\ \varepsilon_s & \text{for all the others,} \end{cases} \tag{10}$$

and the constant C is equal to pG, where p is the number of pairs of nucleons with quantum numbers $s+$, $s-$. Studying the second variation of the mean value of the Hamiltonian, we find that it is a positive definite form. Thus, this simple solution of the Hartree-Fock equations is stable[1]. The Hamiltonian (9) with the separated self-consistent field has the form

$$H = H_1 + H_2,$$

$$H_1 = \sum_s E_s \left(a^+_{s+} a_{s+} + a^+_{s-} a_{s-}\right) + C,$$

$$H_2 = -G \sum_{s,s'} a^+_{s+} a^+_{s-} a_{s'-} a_{s'+} + G \sum_s{}' \left(a^+_{s+} a_{s+} + a^+_{s-} a_{s-}\right) - C. \tag{11}$$

In the second sum in H_2 one must sum over the levels on which there are two nucleons.

Let us note that if one takes the more general interaction

$$-G \sum_{s,s'} V_{ss'} a^+_{s+} a^+_{s-} a_{s'-} a_{s'+}, \tag{12}$$

then we obtain the same result with the equation for the energy levels in the self-consistent field:

$$E_s = \begin{cases} \varepsilon_s - G V_{ss} & \text{for levels } s \text{ on which there are two nucleons,} \\ \varepsilon_s & \text{for all the others.} \end{cases} \tag{13}$$

Recently it has become clear that many properties of even-even nuclei can be explained on the basis of a model of independent particles with pairing interaction as the residual interaction between nucleons (the superconducting model of the nucleus)[2]. In this model the one-particle energy levels are calculated from experimental data for neighboring odd nuclei and are regarded as levels in the complete self-consistent field. Consequently, we arrive at the conclusion that one must solve the problem with the Hamiltonian (11), where the energy levels E_s are specified and where the self-consistent field has been completely separated from the interaction. If we wish to use the Hamiltonian in the form (9), which is more convenient and as is usually done, then from the known E_s we must calculate ε_s from formula (10). Let us note that this point of view differs from the one usually adopted; see, for example, [2], where each energy level is shifted by an amount depending on s.

In conclusion I express my gratitude to V. Rybarskaya for valuable discussions and to V. G. Solov' ev for the discussion.

Joint Institute
for Nuclear Research

Received
9 II 1962

REFERENCES

[1] N. N. Bogolyubov, DAN, **119**, 52 (1959).
[2] V. G. Solov' ev, *Pair correlations of the superconducting type in atomic nuclei*, Doctoral dissertation, Preprint of the Joint Institute for Nuclear Research, R-801, Dubna, 1961.

Note: Figure translations are in progress. See original paper for figures.