Theory of Elasticity

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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ELEMENTARY ELASTO-PLASTIC DEFORMATION IN TORSION OF A ROD WITH A SMALL LONGITUDINAL GROOVE ON THE SURFACE

Assuming that the material of the rod is strain-softening ([1]), the problem is considered of determining the stress-strain state in the neighborhood of an elementary plastic shear under torsion of a rod with a small semicylindrical groove on the surface.

We assume that the radius of the groove is small in comparison with the transverse dimensions of the rod and that the contour of the cross section in the neighborhood of the recess may be regarded as rectilinear, i.e., the body of the rod is considered as a half-space with a semicylindrical groove on the surface (Fig. 1).

Before plastic shear deformation arises under torsion, the maximum tangential stresses act at the bottom of the recess at points of the line AB (Fig. 1). When $\tau_{\max}$ reaches the upper yield limit τ_m, plastic shear occurs through the indicated line along the normal to the contour. The depth of shear h is determined from the condition of stress boundedness.

The discontinuous shear deformation occurring on the slip surfaces may be considered ([2]) as the result of the presence of screw dislocations distributed with density $\mu(x)$ along the segment $[a, a+h]$ of the Ox axis.

Fig. 1

The stresses and displacements caused in a half-space with a semicylindrical groove by a system of screw dislocations distributed in the indicated manner can be determined, respectively, by the formulas

$$\tau_{xz}^{(1)} - i\tau_{yz}^{(1)} = \frac{G}{2\pi i}\int\limits_L \frac{\nu(s)}{\zeta - s} \quad \left(\sigma_x^{(1)} = \sigma_y^{(1)} = \sigma_z^{(1)} = \tau_{xy}^{(1)} = 0,\ \zeta = x + iy\right), \quad (1)$$

$$w^{(1)}(x, y) = \operatorname{Re}\left\{\frac{1}{2\pi i}\int\limits_L \frac{\chi(s)\,ds}{s - \zeta}\right\} \quad \left(u^{(1)} = v^{(1)} = 0\right), \quad (2)$$

where the contour of integration L consists of two segments $[-n, -m]$, $[m, n]$ of the Ox axis, with $n = a + h$, $m = a^2/n$, and the functions $\nu(s)$ and $\chi(s)$ are determined by the equalities

$$\nu(s) = \begin{cases} \mu(s), & (a \le s \le n), \\ -\dfrac{a^2}{s^2}\mu\left(\dfrac{a^2}{s}\right), & (m \le s \le a), \end{cases} \qquad \nu(-s) = -\nu(s); \quad (3)$$

$$\chi(s) = \int\limits_{-n}^{s} \nu(\sigma)\,d\sigma. \quad (4)$$

On the slip surfaces the shear stresses are equal in magnitude to the constant τ_c, called the lower yield point, and are directed opposite to the direction of shear, i.e., the condition holds

$$\tau_{yz}^0(x, 0) + \tau_{yz}^{(1)}(x, 0) = \tau_c \qquad (a \le x \le a + h), \quad (5)$$

where τ_{yz}^0 is the stress determined without taking plastic deformation into account.

If the stress $\tau_{yz}^{(1)}(x, 0)$ is expressed by formula (1) and substituted into condition (5), then we obtain the following equation for determining the function $v(s)$:

$$\frac{1}{\pi}\int_L \frac{v(s)}{s - x} = f(x), \quad (6)$$

where we have put

$$f(x) = \begin{cases} \dfrac{2}{G}\left[\tau_{yz}^0(x, 0) - \tau_c\right], & (a \le x \le n), \\ \dfrac{2}{G}\dfrac{a^2}{x^2}\left[\tau_{yz}^0\left(\dfrac{a^2}{x}, 0\right) - \tau_c\right], & (m \le x \le a), \end{cases} \qquad f(-x) = f(x). \quad (7)$$

Fig. 2

Figure 2: Fig. 2

From the condition of boundedness of the stresses, by virtue of formula (1), it follows that the function $v(s)$ is bounded. A bounded solution of equation (6) exists ([3]) under the condition

$$\int_L \frac{s^k f(s)\,ds}{\sqrt{(n^2-s^2)(s^2-m^2)}} = 0 \qquad (k=0,1) \tag{8}$$

and is given by the formula

$$v(x) = \frac{1}{\pi}\sqrt{(n^2-x^2)(x^2-m^2)} \int_L \frac{f(s)\,ds}{\sqrt{(n^2-s^2)(s^2-m^2)}(x-s)}. \tag{9}$$

Substituting expression (9) into formula (1), for the additional shear stresses caused by plastic shear, we finally obtain

$$\tau_{xz}^{(1)} - i\tau_{yz}^{(1)} = \frac{G}{2\pi i}\sqrt{(\zeta^2-n^2)(\zeta^2-m^2)} \int_L \frac{f(s)\,ds}{\sqrt{(n^2-s^2)(s^2-m^2)}(s-\zeta)}, \tag{10}$$

where by the radical

$$\sqrt{R(\zeta)} = \sqrt{(\zeta^2-n^2)(\zeta^2-m^2)}$$

we mean the branch holomorphic in the plane $\zeta = x + iy$ with a cut along L, whose expansion in a neighborhood of the infinitely distant point in descending powers of ζ has the form

$$\sqrt{R(\zeta)} = \zeta^2 - \frac{n^2+m^2}{2} - \frac{1}{8}\frac{(n^2-m^2)^2}{\zeta^2} - \cdots. \tag{11}$$

Fig. 2

For the torsion of a prismatic bar with a shallow semicylindrical groove on the surface, for $\tau_{yz}^0(x,0)$, using Weber’ s solution for the profile ([4]), we can obtain

$$\tau_{yz}^0(x,0) = \frac{\alpha\tau^m}{2}\left(1+\frac{a^2}{x^2}\right) \tag{12}$$

$$(\alpha > 1, \quad a \le x \le n).$$

If the function $f(x)$, determined with allowance for equalities (7) and (12), is substituted into condition (8), then after evaluating the integrals we obtain the following

equation for determining the depth of the plastic shear h:

$$2q^2(p-1)F(k,\varphi) - 2E(k,\varphi) + pE\left(k, \frac{\pi}{2}\right) = 0, \tag{13}$$

where $p = \dfrac{\alpha\tau_m}{\tau_c}$, $q = \dfrac{a}{a+h}$; F, E are elliptic integrals of the first and second kind, respectively, and $k = \sqrt{1-q^4}$ and $a = \operatorname{arc\,ctg} q$ are their arguments.

The graph of the dependence of the quantities p and q, constructed on the basis of formula (13), is shown in Fig. 2.

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Note: Figure translations are in progress. See original paper for figures.