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Abstract

Full Text

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On the Phase Motion of Particles in Accelerators with Fluctuating Parameters

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The equation of phase oscillations of a particle in accelerators whose parameters undergo small random perturbations can, in most cases, be written, restricting oneself to the linear approximation, in the following form (see, for example, ([1])):

$$\frac{d^2q}{dt^2} + \omega^2(t)q = a(t)\psi(t), \tag{1}$$

where q is the distance between the particle under consideration and the "ideal" synchronous particle at the moment when the latter passes through the middle of the accelerating gap; ψ is a random quantity characterizing the deviation of the phase of the synchronous particle from the "ideal" one as a result of perturbations of various types of parameters (for example, the lengths of the accelerating gaps, the voltage and frequency of the accelerating field, etc.); $\omega^2(t)$, $a(t)$ are known slowly varying functions of time, expressed in dimensionless units t/T (T is the period of the high-frequency oscillations of the accelerating field).

Let us introduce, along with (1), the equation

$$\frac{d^2q_0}{dt} + \omega^2(t)q_0 = 0, \tag{2}$$

which describes the phase oscillations of a particle in an "ideal" accelerator, and the quantities $Q = q - q_0$, $\dot{Q} = \dot{q} - \dot{q}_0$, characterizing the deviations of the position and velocity of the particle from the "ideal" ones.

The problem arises of finding the probability density $\Phi(t, Q, \dot{Q})$. If one assumes that $\overline{\psi(t)} = 0$, $\overline{\psi(t)\psi(t')} = \psi_0^2\delta(t - t')$ (i.e., that the perturbations $\psi(t)$ at different instants of time are statistically independent, and their mean-square value is the same for all t and equal to ψ_0^2) and that the solutions of equations (1), (2) can be found (in view of the slowness of variation of the functions $\omega^2(t)$, $a(t)$) in the WKB approximation, i.e.,

$$q_0 \sim \omega^{-1/2} \cos\left(\int^t \omega\, dt\right),$$

then, as can be shown by the Fokker–Planck method, for Φ one obtains an equation of diffusion type

$$\frac{\partial \Phi}{\partial t} + \dot{Q}\,\frac{\partial \Phi}{\partial Q} - \omega^2 Q\,\frac{\partial \Phi}{\partial \dot{Q}} = \nu(t)\left(\frac{\partial^2 \Phi}{\partial Q^2} + \omega^2(t)\frac{\partial^2 \Phi}{\partial \dot{Q}^2}\right), \tag{3}$$

where

$$\nu(t) = \frac{\psi_0^2}{2\omega(t)}\left(\frac{a^2(t)}{\omega(t)} + \omega^2(t)\int_{t_0}^{t}\frac{a^2(\tau)}{\omega(\tau)}\,d\tau\right).$$

The function Φ satisfies at the initial time t_0 the condition

$$\Phi(t_0, Q, \dot{Q}) = \delta(Q)\delta(\dot{Q}), \tag{4}$$

which has the meaning that at $t = t_0$ the statistical spread is still absent.

With the aid of the integral transformation

$$\Phi = \int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\Psi(t,\xi,\eta)e^{\xi Q + \eta\dot{Q}}\,d\xi\,d\eta \tag{5}$$

equation (3) reduces to a first-order equation

$$\frac{\partial \Psi}{\partial t} - \eta\frac{\partial \Psi}{\partial \xi} + \xi\omega^2(t)\frac{\partial \Psi}{\partial \eta} - \nu(t)(\eta^2 + \omega^2(t)\xi^2)\Psi = 0 \tag{6}$$

with the additional condition (following from (4)):

$$\psi(t_0, \xi, \eta) = -\frac{1}{4\pi^2}. \tag{7}$$

The solution of (6) can be found by the usual method of characteristics and leads to the formula

$$\Psi = -\frac{1}{4\pi^2}\exp\left[-\left(\omega\xi^2 + \frac{\eta^2}{\omega}\right)\int_{t_0}^{t}\nu\omega\,dt\right], \tag{8}$$

whence, using (5), we finally find:

$$\Phi = \frac{1}{4\pi \int_{t_0}^{t} \nu\omega \, dt} \exp\left(-\frac{Q^2}{\frac{4}{\omega} \int_{t_0}^{t} \nu\omega \, dt} - \frac{\dot{Q}^2}{4\omega \int_{t_0}^{t} \nu\omega \, dt} \right), \tag{9}$$

In conclusion it should be emphasized that the distribution (9) is in fact valid for any dynamical system characterized by a slowly time-varying potential $\frac{1}{2}\omega^2(t)q^2$ and subject to the action of a random force $a(t)\psi(t)$.

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CITED LITERATURE

[1] S. Livingston, *Accelerators*, IL, 1956.

Note: Figure translations are in progress. See original paper for figures.