B. A. Al' tshul'

In paper (1) a method of asymptotic approximation was proposed for systems with a rotating phase.

1962

SovietRxiv

Abstract

Full Text

B. A. Al' tshul'

APPLICATION OF THE METHOD OF ASYMPTOTIC APPROXIMATION TO THE STUDY OF THE DYNAMICS OF THE BOUSSINESQ-SARDEAU REGULATOR

(Presented by Academician N. N. Bogolyubov, 20 XI 1961)

In paper (1) a method of asymptotic approximation was proposed for systems with a rotating phase.

Consider the system of equations

$$\frac{dx}{dt} = X_k(x_1, \dots, x_r, \theta) \qquad (k = 1, \dots, r),$$

$$\frac{d\theta}{dt} = \lambda\omega(x_1, \dots, x_r) + A(x_1, \dots, x_r, \theta), \tag{1}$$

where λ is a large parameter; $X_k(x_1, \dots, x_r, \theta)$, $A(x_1, \dots, x_r, \theta)$ are periodic functions of θ with period 2π.

The variable θ can be eliminated from the right-hand sides of equations (1) with any desired degree of accuracy in an expansion in powers of $\frac{1}{\lambda}$. In particular, it is shown that, by means of the change of variables

$$x_k = \bar{x}_k + \frac{1}{\lambda}\sum_{n=1}^{\infty}\frac{1}{n\omega}\left(-G_{k,n}\cos n\bar{\theta} + F_{k,n}\sin n\bar{\theta}\right) + o\left(\frac{1}{\lambda^2}\right),$$

$$\theta = \bar{\theta} + \frac{1}{\lambda}\sum_{n=1}^{\infty}\frac{1}{n\omega}\left(-g_n\cos n\bar{\theta} + f_n\sin n\bar{\theta}\right) -$$

$$-\frac{1}{\lambda}\sum_{\substack{n,q\\ n\neq 0}}\frac{1}{n^2\omega^2}\frac{\partial\omega}{\partial x_q}\left(F_{q,n}\cos n\bar{\theta} + G_{q,n}\sin n\bar{\theta}\right) + o\left(\frac{1}{\lambda^2}\right) \tag{2}$$

one can obtain the system of equations

$$\frac{d\bar{x}_k}{dt} = X_{k,0} - \frac{1}{\lambda}\sum_{n=1}^{\infty}\frac{1}{2\omega}\left(F_{k,n}f_n + G_{k,n}g_n\right) -$$

$$-\frac{1}{\lambda}\sum_{\substack{n,q\\ n\neq 0}}\frac{1}{2n\omega}\left(\frac{\partial F_{k,n}}{\partial x_q}G_{q,n}-\frac{\partial G_{k,n}}{\partial x_q}F_{q,n}\right)+$$

$$+\frac{1}{\lambda}\sum_{\substack{q,n\\ n\neq 0}}\frac{1}{2\omega^2 n}\frac{\partial \omega}{\partial x_q}\left(F_{k,n}G_{q,n}-G_{k,n}F_{q,n}\right)+O\left(\frac{1}{\lambda^2}\right), \tag{3}$$

$$\frac{d\bar{\theta}}{dt}=\lambda\omega+A_0+\frac{1}{\lambda}\sum_{\substack{n,p,q\\ n\neq 0}}\frac{1}{4n^2\omega^2}\frac{\partial^2\omega}{\partial x_p\,\partial x_q}\left(F_{p,n}F_{q,n}+G_{p,n}G_{q,n}\right)-$$

$$-\frac{1}{\lambda}\sum_{\substack{n,q\\ n\neq 0}}\frac{1}{2\omega^2 n}\frac{\partial \omega}{\partial x_q}\left(g_{nF_{q,n}}-f_{nG_{q,n}}\right)+\frac{1}{\lambda}\sum_{\substack{n,q\\ n\neq 0}}\frac{1}{2\omega n}\left(\frac{\partial g_n}{\partial x_q}F_{q,n}-\frac{\partial f_n}{\partial x_q}G_{q,n}\right)-$$

$$-\frac{1}{\lambda}\sum_{n}\frac{1}{2\omega}\left(f_n^2+g_n^2\right)+O\left(\frac{1}{\lambda^2}\right),$$

which gives a solution of the posed problem with accuracy up to quantities of order $\dfrac{1}{\lambda^2}$.

The right-hand sides of systems (2) and (3) are expressed in terms of the Fourier coefficients of the functions X_k and A:

$$X_k=X_{k,0}+\sum_{n=1}^{\infty}\left(F_{k,n}\cos n\bar{\theta}+G_{k,n}\sin n\bar{\theta}\right),$$

$$A=A_0+\sum_{n=1}^{\infty}\left(f_n\cos n\bar{\theta}+g_n\sin n\bar{\theta}\right).$$

We shall apply the proposed method to the study of the dynamics of the Buassa-Sardu regulator, which is used to ensure uniform lowering of a load suspended on a cable. A description of this regulator is given in paper ([2]), where, by the Lyapunov-Poincaré methods, the conditions for the existence and stability of a solution corresponding to lowering of the load with a constant mean angular velocity ω over a period are established.

The operation of the regulator is described by the following system of equations:

$$I\ddot{\theta}+k\dot{\theta}=Mgr+\mu h(x-h\sin\theta)\cos\theta,$$

$$m\ddot{x} + \beta\dot{x} + \mu x = mg + \mu h \sin\theta, \tag{4}$$

where $\theta = \theta(t)$ and $x = x(t)$ are functions of time t, and all the other parameters entering system (4) are constants.

In order to reduce system (4) to the form (1), we introduce the following change of variables:

$$x = x_1 + \frac{mg}{\mu} + \mu h \frac{(\mu - m\omega^2)\sin\theta - \beta\omega\cos\theta}{(\mu - m\omega^2)^2 + \beta^2\omega^2},$$

$$\dot{x} = x_2 + \mu h\omega \frac{(\mu - m\omega^2)\cos\theta + \beta\omega\sin\theta}{(\mu - m\omega^2)^2 + \beta^2\omega^2}, \tag{5}$$

$$\dot{\theta} = \omega + x_3,$$

where ω is the sought value of the mean angular velocity of the shaft of the winch over a period.

Introduce the following notation:

$$\mu - m\omega^2 = a_1, \qquad \beta\omega = a_2. \tag{6}$$

Substituting (5) into (4) and taking into account the notation (6), we obtain

$$\frac{dx_1}{dt} = x_2 - \mu h \frac{a_1\cos\theta + a_2\sin\theta}{a_1^2 + a_2^2} x_3,$$

$$\frac{dx_2}{dt} = \frac{\mu h}{m}\sin\theta - \frac{\mu h\omega}{a_1^2 + a_2^2}(a_2\cos\theta - a_1\sin\theta)(x_3 + \omega) -$$

$$-\frac{\beta x_2}{m} - \frac{\beta\mu h\omega}{a_1^2 + a_2^2}(a_1\cos\theta + a_2\sin\theta) - \frac{\mu x_1}{m}$$

$$-\frac{\mu^2 h}{m(a_1^2 + a_2^2)}(a_1\sin\theta - a_2\cos\theta), \tag{7}$$

$$\frac{dx_3}{dt} = \frac{Mgr}{I} + \frac{\mu h x_1 + mgh}{I}\cos\theta + \frac{\mu^2 h^2}{I(a_1^2 + a_2^2)}(a_1\sin\theta - a_2\cos\theta)\cos\theta -$$

$$-\frac{k\omega}{I} - \frac{kx_3}{I} - \frac{\mu h^2}{I}\sin\theta\cos\theta,$$

$$\frac{d\theta}{dt} = \omega + x_3.$$

The asymptotic averaging method can now be applied directly to equations (7).

Carrying out the corresponding calculations, with the aid of a substitution of the form (2)

$$x_1 = \bar{x}_1 + \frac{1}{\omega}\left[\frac{\mu h x_3}{a_1^2 + a_2^2}\left(a_2 \cos\bar{\theta} - a_1 \sin\bar{\theta}\right)\right] + o\left(\frac{1}{\omega^2}\right),$$

$$x_2 = \bar{x}_2 + \frac{1}{\omega}\left\{\left[-\frac{\mu h}{\omega} + \frac{\mu h}{a_1^2 + a_2^2}\left(a_1\omega^2 + a_1\omega x_3\right) - \frac{\beta\omega a_2}{m} - \frac{\mu a_1}{m}\right]\cos\bar{\theta} - \right.$$

$$\left. - \frac{\mu h}{a_1^2 + a_2^2}\beta\omega^2 \bar{x}_3 \sin\bar{\theta}\right\} + o\left(\frac{1}{\omega^2}\right), \tag{8}$$

$$x_3 = \bar{x}_3 + \frac{1}{\omega}\left(\frac{\mu h x_1}{I} + \frac{mgh}{I}\right)\sin\bar{\theta} + \frac{1}{2\omega}\left[\left(\frac{\mu h^2}{2I} - \frac{\mu^2 h^2 a_1}{2I\left(a_1^2 + a_2^2\right)}\right)\cos 2\bar{\theta} - \right.$$

$$\left. - \frac{\mu^2 h^2}{2I\left(a_1^2 + a_2^2\right)} a_2 \sin 2\bar{\theta}\right] + o\left(\frac{1}{\omega^2}\right),$$

$$\theta = \bar{\theta} + o\left(\frac{1}{\omega^2}\right)$$

we obtain a system of equations of the form (3):

$$\frac{d\bar{x}_1}{dt} = \bar{x}_2 - \frac{\mu h^2 a_2}{2I\omega\left(a_1^2 + a_2^2\right)}\left(\mu\bar{x}_1 + mg\right),$$

$$\frac{d\bar{x}_2}{dt} = -\frac{\beta\bar{x}_2}{m} - \frac{\mu\bar{x}_1}{m} - \frac{\mu h^2 a_1}{2I\left(a_1^2 + a_2^2\right)}\left(\mu\bar{x}_1 + mg\right), \tag{9}$$

$$\frac{d\bar{x}_3}{dt} = \frac{Mgr}{I} - \frac{\mu^2 h^2 a_2}{2I\left(a_1^2 + a_2^2\right)} - \frac{k\omega}{I} - \frac{kx_3}{I} + \frac{\mu^2 h^2 a_3 \bar{x}_3}{2I\omega\left(a_1^2 + a_2^2\right)},$$

$$\frac{d\bar{\theta}}{dt} = \omega + \bar{x}_3.$$

Thus, the problem has been reduced to the integration of equations (9), which, as is evident, presents no particular difficulties.

However, even without integrating equations (9), one can obtain certain relations for the dynamics of the Boassa–Sardu governor, for example, a formula relating the weight of the descending load and other parameters of the system to ω when the load is lowered with a constant mean velocity over a period, $\dot{\bar{\theta}} = \omega$.

As we see from the last equation of system (9), this will hold when $\bar{x}_3 \equiv 0$. But if $\bar{x}_3 \equiv 0$, then also $\dot{\bar{x}}_3 \equiv 0$. Therefore, as is evident from the third equation of system (9), the condition for lowering the load with a constant mean velocity over a period will be

$$\frac{Mgr}{I} - \frac{\mu^2 h^2 a_2}{2I\left(a_1^2 + a_2^2\right)} - \frac{k\omega}{I} = 0,$$

whence, taking (6) into account, we obtain

$$\frac{Mgr - k\omega}{\mu h^2} = \frac{\mu\beta\omega}{2\left[(\mu - m\omega^2)^2 + \beta^2\omega^2\right]},$$

a formula playing the principal role in the calculation of the governor.

In conclusion, let us note that the equations (9) obtained by us, together with the substitution (8), make it possible to study not only the stationary regime, but also to determine approximately the transient process.

Received
20 X 1961

REFERENCES CITED

1. N. N. Bogoliubov, D. N. Zubarev, *Ukr. Mat. Zhurn.*, **7**, No. 1, 5 (1955).

2. I. I. Blekhman, G. Yu. Dzhanelidze, *Izv. AN SSSR, OTN*, No. 10, 48 (1955).

Note: Figure translations are in progress. See original paper for figures.