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Abstract

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THEORY OF THE DEPOSITION OF HIGHLY DISPERSE AEROSOLS FROM A FLOW ON AN ULTRATHIN CYLINDER

(Presented by Academician M. M. Dubinin, 30 III 1962)

To solve problems connected with the filtration of aerosols, it becomes necessary to give a theoretical analysis of the process of deposition of highly disperse aerosols from a flow on ultrathin stationary cylinders, whose diameters are of the order of, or smaller than, the mean free path of gas molecules. Until recently this problem had not been solved, although the question of the resistance of such cylinders to a gas flow was discussed in ([1]).

In a first approximation the calculation can be carried out if one uses the equation of convective diffusion at small flow velocities and considers the simplest case of a cylinder so thin that it does not disturb the flow and its diameter is smaller than the diameter of the particle, as a result of which the hydrodynamic forces are negligibly small. To solve the problem it is expedient to introduce the concept of a jump in concentration near the cylinder and to assume that, for the particles, the law of uniform distribution of energy over the degrees of freedom is obeyed. Under these conditions the characteristic parameters determining the behavior of a particle near the cylinder are its mean "thermal velocity"

$$v_T = \sqrt{\frac{8kT}{\pi m}}$$

and the "apparent mean free path" δ of the particle, introduced by Smoluchowski ([2]) and related to the relaxation time τ by the relation

$$\delta = v_T\tau = v_T\frac{m}{B},$$

where B is the mobility of the particle.

Convective diffusion for particles of radius r when an aerosol flows past a stationary cylinder of radius R with velocity v_0 perpendicular to its axis is described for the stationary case by the equation

$$D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right)-v_0\frac{\partial c}{\partial x}=0, \tag{1}$$

where $\rho=\sqrt{x^2+y^2}$, and it is assumed that $c=c_0$ when $\rho\to\infty$.

The boundary condition is obtained by "matching" the diffusion-convective and molecular fluxes, when the first of them has a value for distances from $\rho=\infty$ to $\rho=\rho_0$ from the axis, if $\rho_0=r+R+\delta$, and the second up to the distance $\rho_1\le\rho\le\rho_0$, if $\rho_1=r+R$. This condition has the form

$$\left|D\frac{\partial c}{\partial \rho}-cv_0\cos\varphi\right|_{\rho=\rho_0} L\rho_0\,d\varphi$$
$$=\left|\frac{cv_T}{4}-\frac{cv_0\cos\varphi}{2}\left[1+\operatorname{erf}\left(\frac{v_0}{v_T}\cos\varphi\right)\right]\right|_{\rho=\rho_1} L\rho_1\,d\varphi \tag{2}$$

within the limits of the azimuthal angle $d\varphi$ for a cylinder length L.

The simplest case corresponds to the condition $v_T\gg v_0$, when the quantity $\operatorname{erf}\left(\frac{v_0}{v_T}\cos\varphi\right)$ can be neglected and the hydrodynamic forces, which are very small if $r\gg R$, need not be taken into account.

The solution of equation (1) with condition (2) in this particular case has the form

$$\frac{c}{c_0}=1-e^{a\rho\cos\varphi}\sum_{n=0}^{\infty}B_n(a\rho_0)K_n(a\rho)\cos n\varphi, \tag{3}$$

where $K_n(a\rho)$ is a Bessel function of the second kind of imaginary argument of order n, and the values of the coefficients $B_n(a\rho_0)$ can be found from the boundary condition (2). Here

$$a=\frac{v_0}{2D}\frac{r+R+2\delta}{r+R+\delta}.$$

If we set

$$[\mathrm{Pe}]=z=a\rho_0=\frac{v_0}{2D}(r+R+2\delta)$$

equal to the modified Peclet number and introduce the parameter

$$p = \frac{v_T}{2v_0}\frac{r+R}{r+R+2\delta},$$

then it can be shown that

$$B_0 = \frac{pI_0(z) - I_1(z)}{pK_0(z) + K_1(z)},$$

where $I_0(z)$ and $I_1(z)$ are Bessel functions of the first kind of imaginary argument of zero and first orders, and $K_0(z)$ and $K_1(z)$ are Bessel functions of the second kind of the same orders. For $n \geq 1$ in the sum (3) we have

$$B_n = 2\,\frac{\alpha_n I_0(z) - \beta_n I_1(z)}{\alpha_n K_0(z) + \beta_n K_1(z)},$$

where the values of α_n and β_n are found by applying the recurrence formulas for Bessel functions

$$\begin{aligned} &\alpha_1 = 1; && \beta_1 = p + u; \\ &\alpha_2 = p + 2u; && \beta_2 = 1 + 2pu + 4u^2; \\ &\alpha_3 = 1 + 4pu + 12u^2; && \beta_3 = p + 5u + 8pu^2 + 24u^3, \end{aligned}$$

where $u = 1/z$ has been set. It can be shown that, with the found values of the coefficients $B_n(a\rho_0)$ taken into account, the series in formula (3) converges. Relation (3) gives the distribution of the aerosol concentration near the cylinder at points corresponding to the coordinates φ and ρ, or $c = f(\varphi, \rho)$.

Solution (3) makes it possible to calculate the number of particles dv_φ deposited during the time dt on the cylinder as the flux of them through the surface $\rho_0 L\, d\varphi$, i.e.,

$$dv_\varphi = D\left|D\frac{\partial c}{\partial \rho} - cv_0\cos\varphi\right|_{\rho=\rho_0} L\rho_0\, d\varphi\, dt.$$

The total flux of particles is obtained by integrating the expression written above over all angles from 0 to 2π, and in this way we obtain the efficiency of aerosol deposition on the cylinder, i.e.,

$$\eta = \frac{dv/dt}{2\rho_0 c_0 v_0 L} = \frac{1}{4}\sum_{n=0}^{\infty} B_n(z)\left[K_n(z)\int_0^{2\pi} e^{a\rho_0\cos\varphi}\cos n\varphi\cos\varphi\, d\varphi - K_n'(z)\int_0^{2\pi} e^{a\rho_0\cos\varphi}\cos n\varphi\, d\varphi\right].$$

Using the known integral representation of the functions $I_n(z)$ and $I_n'(z)$, we obtain:

$$\eta = \frac{\pi}{2}\sum_{n=0}^{\infty} B_n(z)\,[K_n(z)I'_n(z) - K'_n(z)I_n(z)] = \frac{\pi}{2}\sum_{n=0}^{\infty} S_n(z). \tag{4}$$

The series converges so rapidly that, with sufficient approximation, it is permissible to restrict ourselves to the first term S_0, which we obtain by applying recurrence formulas for the Bessel functions and their derivatives:

$$S_0 = \frac{pI_0(z) - I_1(z)}{pK_0(z) + K_1(z)}\,[I_1(z)K_0(z) + I_0(z)K_1(z)]. \tag{5}$$

According to the written values of a, p, and ρ_0, η can be calculated with the required accuracy, but under the conditions $v_T \gg v_0$ and $r \gg R$. The results of such calculations reduce to the fact that, for small particles at $10^{-6} \leq r \leq 10^{-5}$ cm and ultrathin cylinders with $10^{-6} \leq R \leq 4 \cdot 10^{-6}$ cm, at small flow velocities from 0.5 to 1.0 cm/sec, the deposition efficiency $\eta > 1$; moreover, as r increases it decreases monotonically and diminishes with increasing velocity. Thus, this theory describes the deposition process in the diffusion region. Comparison with the few available experimental data ([3]) showed that the theoretical values of η are somewhat greater than the experimental ones. Calculations for comparison with experimental data are inadmissible for particles with $r > 1 \cdot 10^{-5}$ cm, since in this region the process of particles being blown away by the flow plays a noticeable role. The reasons for the discrepancy between the theoretical values of η and the experimental ones are still unclear. It is possible that it is connected with the quantity δ, which is introduced into the theory in a not strictly rigorous manner.

A characteristic feature of the process under consideration is that the radius of the cylinder enters the boundary condition through the parameter ρ_0, and therefore, as $R \to 0$, the problem admits a physical solution for a finite value of η, i.e., it will refer to the deposition of particles on a "line," which corresponds to the primitive calculation carried out in work ([4]). Thus, the diameter of the cylinder is not the determining parameter here. This is indirectly consistent with the experimental data ([3]), where the deposition efficiency on thin cylinders depended only weakly on the cylinder diameter.

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Note: Figure translations are in progress. See original paper for figures.