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Abstract

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POST-CRITICAL DEFORMATIONS OF CYLINDRICAL SHELLS UNDER TORSION

In the present note we set forth some results of an investigation of the post-critical elastic state of cylindrical shells under torsion. The case of axial compression and external pressure has been considered in the author' s works (1).

By post-critical we mean such elastic deformations in which the shape of the shell differs significantly from the initial one. The transition to such deformations occurs as a result of loss of stability of the basic shape and is accompanied by a decrease in the load carried by the shell to a certain value s_i, called the lower critical load. The load s_e at which loss of stability occurs is called the upper critical load.

A number of theoretical and experimental studies have been devoted to the investigation of the post-critical elastic state of a cylindrical shell under torsion, in particular to the determination of the magnitude of the critical loads (see the bibliography in the book (2)). Our investigation of the question differs essentially in method from the investigations of other authors. The basis of this method is formed by three simple and natural propositions.

1. The post-critical elastic deformation of a shell is, in essence, an isometric bending.
2. The periodicity of the structure of the shell shape in post-critical deformation is predetermined by the character of the periodicity of the deflections on it at the moment of loss of stability.
3. The transition to post-critical deformations of the shell is associated with the appearance of sharply expressed ridges on its surface. The energy of elastic deformation of the shell admits a natural division into two parts, U_G and U_γ; U_γ is the energy of elastic deformation caused by strong local bending along the ridges, while U_G is the energy of bending over the basic surface.

The investigation of the post-critical elastic state of a cylindrical shell under torsion, based on the three propositions indicated, consists, in general outline, of the following. First of all, we specify a class of surfaces Z, isometric to the cylinder, containing the shape of the shell in post-critical deformation, under the assumption that the latter is an isometric bending. Each surface of this class

Fig. 1

Figure 1: Fig. 1

consists of a number of congruent, symmetrically arranged general cylindrical surfaces separated by ridges (Fig. 1).

The surfaces Z of the constructed class depend on an integer parameter n, determining the periodicity of the shape of the surface, and on a function of one variable $y(x)$, determining the shape of the surface in the small. Finding the shape of the shell in the class of surfaces Z reduces to determining the parameter n and the function $y(x)$.

We determine the parameter n by identifying the periodicity of the structure of the surface Z with the periodicity of the deflections on the surface of the shell at the moment of loss of stability. We determine the function $y(x)$ from energy considerations, namely from the condition of minimum energy for a given total deformation—the angle of twist.

After the shape of the shell surface under postcritical deformation has been determined, the elastic-strain energy of the shell becomes a known function of the angle of twist. Now, comparing the elementary work performed by the external load and the change in elastic-strain energy, we establish the relation between the deformation and the load sustained. Minimizing the latter with respect to the deformation parameter, we find the lower critical load.

The main results of the study may be formulated as follows.

1. A hinged cylindrical shell of radius R, length L, and thickness δ, under the action of a tangential load s uniformly distributed along the edge, loses stability at

$$s_e \simeq 0.8E\frac{\delta}{R}\left(\frac{R\delta}{L^2}\right)^{1/4}.$$

Fig. 1

2. As a result of the loss of stability, upon transition to postcritical deformations the load sustained by the shell decreases, reaches a certain minimum s_i, and then increases again. The transition to postcritical deformations under the action of a load causing loss of stability occurs with a "snap-through."

3. If the geometrical dimensions and mechanical characteristics of the shell material satisfy the condition

$$3.6E\frac{\delta}{R} < \sigma_{\text{B}}$$

(E is the modulus of elasticity, σ_{B} is the ultimate strength), then the smallest load sustained by the shell under postcritical deformation, i.e., the lower critical load,

$$s_i \simeq 0.2E\frac{\delta}{R}\left(\frac{R\delta}{L^2}\right)^{1/4},$$

and thus amounts to one quarter of the upper critical load.

An analogous study was carried out for the case of combined loading of a cylindrical shell—torsion and axial compression.

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2. A. S. Vol' mir, *Flexible Plates and Shells*, 1956.

Note: Figure translations are in progress. See original paper for figures.