# ELECTRICAL ENGINEERING

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**Abstract**

**Full Text**

*ELECTRICAL ENGINEERING*

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# A NEW METHOD FOR THE CALCULATION OF MAGNETIC INTERFERENCE

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Measurements and the practical use of magnetic fields from moving platforms are substantially complicated by rotational interference. This interference, which consists of changes in the Earth' s magnetic field in a coordinate system rigidly connected with the platform, may affect the measuring element both directly and through secondary fields of the platform' s reaction. Below a new method is considered for calculating magnetic interference arising in platforms undergoing small spatial rotation. The results obtained can be used to estimate the magnitude and structure of interference of the indicated type in real platforms, and also in any case when interference associated with small spatial rotation is superposed on the principal regime of the magnetic field.

Consider a platform of arbitrary configuration, rigidly connected with a rectangular coordinate system and performing small spatial oscillations. When the coordinate system is rotated about the axis $x$ through an angle $\Delta\alpha_x$, increments of the projections of the magnetic-field intensity appear in it:

$$\Delta H_{y1} = H'_y - H_y = \mathbf{H}\,(\mathbf{j}' - \mathbf{j}), \qquad \Delta H_{z1} = H'_z - H_z = \mathbf{H}\,(\mathbf{k}' - \mathbf{k}). \tag{1}$$

According to Fig. 1,

$$\Delta H_{y1} = \Delta\alpha_x H_z - \frac{1}{2}\Delta\alpha_x^2 H_y, \qquad \Delta H_{z1} = -\Delta\alpha_x H_y - \frac{1}{2}\Delta\alpha_x^2 H_z. \tag{2}$$

Analogously, we obtain expressions for the increments of the magnetic-field projections $\Delta H_{x2}$ and $\Delta H_{z2}$ upon rotation about the axis $y$, and the increments of the projections $\Delta H_{x3}$ and $\Delta H_{y3}$ upon rotation about the axis $z$:

$$\Delta H_{x2} = -\Delta\alpha_y H_z - \frac{1}{2}\Delta\alpha_y^2 H_x, \qquad \Delta H_{z2} = \Delta\alpha_y H_x - \frac{1}{2}\Delta\alpha_y^2 H_z, \tag{3}$$

$$\Delta H_{x3} = \Delta\alpha_z H_y - \frac{1}{2}\Delta\alpha_z^2 H_x, \qquad \Delta H_{y3} = -\Delta\alpha_z H_x - \frac{1}{2}\Delta\alpha_z^2 H_y. \tag{4}$$

According to (2)–(4), we have

$$\Delta \mathbf{H}' = \Pi \mathbf{H}, \tag{5}$$

where $\Delta \mathbf{H}'$ is the interference vector;

$$\Pi = -\left\{ \begin{array}{ccc} \frac{1}{2}(\Delta\alpha_y^2 + \Delta\alpha_z^2) & -\Delta\alpha_z & \Delta\alpha_y \\ \Delta\alpha_z & \frac{1}{2}(\Delta\alpha_x^2 + \Delta\alpha_z^2) & -\Delta\alpha_x \\ -\Delta\alpha_y & \Delta\alpha_x & \frac{1}{2}(\Delta\alpha_x^2 + \Delta\alpha_y^2) \end{array} \right\}. \tag{6}$$

is a second-rank tensor, which can be regarded as an analogue of an operator composed of Euler coefficients.

Transformation (5) shows that, in problems involving rotating platforms, one can use the solutions of analogous problems for stationary platforms, replacing the field acting on them by the field $\Delta \mathbf{H}'$.

If tensor (6) is represented as the sum of antisymmetric and symmetric components $\Pi = \Pi_1 + \Pi_2$, then

$$\Delta \mathbf{H}' = \vec{\Pi}_1 \times \mathbf{H} + \Pi_2 \mathbf{H}, \tag{7}$$

where $\vec{\Pi}_1 = -(i'\Delta\alpha_x + j'\Delta\alpha_y + k'\Delta\alpha_z)$, or

$$\mathbf{H}' = \mathbf{H} + \vec{\Pi}_1 \times \mathbf{H} + \Pi_2 \mathbf{H}. \tag{8}$$

If, for example, a platform moving in the Earth' s magnetic field oscillates harmonically, then

$$\begin{aligned} \Delta\dot{H}'_{1x} &= \Delta\alpha_{zm} T \cos I_0 \sin A e^{i\psi_z} - \Delta\alpha_{ym} T \sin I_0 e^{i\psi_y}, \\ \Delta\dot{H}'_{1y} &= \Delta\alpha_{xm} T \sin I_0 e^{i\psi_x} - \Delta\alpha_{zm} T \cos I_0 \cos A e^{i\psi_z}, \\ \Delta\dot{H}'_{1z} &= \Delta\alpha_{ym} T \cos I_0 \cos A e^{i\psi_y} - \Delta\alpha_{xm} T \cos I_0 \sin A e^{i\psi_x}; \end{aligned} \tag{9}$$

$$\begin{aligned} \Delta\dot{H}'_{2x} &= T \cos I_0 \cos A \left[\tfrac{1}{4}\Delta\alpha_{ym}^2 e^{i(2\psi_y + \pi/2)} + \tfrac{1}{4}\Delta\alpha_{zm}^2 e^{i(2\psi_z + \pi/2)}\right], \\ \Delta\dot{H}'_{2y} &= T \cos I_0 \sin A \left[\tfrac{1}{4}\Delta\alpha_{xm}^2 e^{i(2\psi_x + \pi/2)} + \tfrac{1}{4}\Delta\alpha_{zm}^2 e^{i(2\psi_z + \pi/2)}\right], \\ \Delta\dot{H}'_{2z} &= T \sin I_0 \left[\tfrac{1}{4}\Delta\alpha_{xm}^2 e^{i(2\psi_x + \pi/2)} + \tfrac{1}{4}\Delta\alpha_{ym}^2 e^{i(2\psi_y + \pi/2)}\right]. \end{aligned} \tag{10}$$

Expressions (9) and (10), respectively for the first and second harmonics, are written in a coordinate system with the $z$-axis directed vertically downward. In these expressions $T$ is the magnitude of the Earth' s field; $I_0$ is the magnetic inclination angle; $A$ is the azimuth, measured from the positive direction of the $x$-axis; $\Delta\alpha_{xm}$, $\Delta\alpha_{ym}$, $\Delta\alpha_{zm}$ and $\psi_x, \psi_y, \psi_z$ are the amplitudes and initial phases of the corresponding platform rotations.

Let us consider the reaction field of a platform made of a nonmagnetic conducting material, arising as a result of eddy currents appearing in it. We introduce scalar magnetic potentials of the reaction field $\varphi_1(x, y, z)$, $\varphi_2(x, y, z)$, and $\varphi_3(x, y, z)$, related to the corresponding components of the disturbance field by the relations $\varphi_1 = f_1(x, y, z)\Delta\dot{H}'_x$, $\varphi_2 = f_2(x, y, z)\Delta\dot{H}'_y$, $\varphi_3 = f_3(x, y, z)\Delta\dot{H}'_z$, where $f_1$, $f_2$, and $f_3$ are functions depending on the properties of the platform. The components of the platform reaction field are expressed by the formulas

$$\dot{H}'_x = \frac{\partial f_1}{\partial x}\Delta\dot{H}'_x + \frac{\partial f_2}{\partial x}\Delta\dot{H}'_y + \frac{\partial f_3}{\partial x}\Delta\dot{H}'_z, \qquad \dot{H}'_y = \frac{\partial f_1}{\partial y}\Delta\dot{H}'_x + \frac{\partial f_2}{\partial y}\Delta\dot{H}'_y + \frac{\partial f_3}{\partial y}\Delta\dot{H}'_z,$$
$$\dot{H}'_z = \frac{\partial f_1}{\partial z}\Delta\dot{H}'_x + \frac{\partial f_2}{\partial z}\Delta\dot{H}'_y + \frac{\partial f_3}{\partial z}\Delta\dot{H}'_z, \tag{11}$$

and, consequently, the vector of the platform reaction field is

$$\dot{\mathbf{H}}' = F\Delta\dot{\mathbf{H}}', \tag{12}$$

where $F$ is a second-rank tensor of the form

$$F = \left\{ \begin{array}{ccc} \partial f_1/\partial x & \partial f_2/\partial x & \partial f_3/\partial x \\ \partial f_1/\partial y & \partial f_2/\partial y & \partial f_3/\partial y \\ \partial f_1/\partial z & \partial f_2/\partial z & \partial f_3/\partial z \end{array} \right\}. \tag{13}$$

On the basis of (5) we have

$$\dot{\mathbf{H}}' = F(\Pi\mathbf{T}) \quad \text{or} \quad \dot{\mathbf{H}}' = K\mathbf{T}, \tag{14}$$

where

$$K = \left\{ \begin{array}{ccc} \frac{\partial f_1}{\partial x}\Pi_{11} + \frac{\partial f_2}{\partial x}\Pi_{21} + \frac{\partial f_3}{\partial x}\Pi_{31} & \frac{\partial f_1}{\partial x}\Pi_{12} + \frac{\partial f_2}{\partial x}\Pi_{22} + \frac{\partial f_3}{\partial x}\Pi_{32} & \frac{\partial f_1}{\partial x}\Pi_{13} + \frac{\partial f_2}{\partial x}\Pi_{23} + \frac{\partial f_3}{\partial x}\Pi_{33} \\ \frac{\partial f_1}{\partial y}\Pi_{11} + \frac{\partial f_2}{\partial y}\Pi_{21} + \frac{\partial f_3}{\partial y}\Pi_{31} & \frac{\partial f_1}{\partial y}\Pi_{12} + \frac{\partial f_2}{\partial y}\Pi_{22} + \frac{\partial f_3}{\partial y}\Pi_{32} & \frac{\partial f_1}{\partial y}\Pi_{13} + \frac{\partial f_2}{\partial y}\Pi_{23} + \frac{\partial f_3}{\partial y}\Pi_{33} \\ \frac{\partial f_1}{\partial z}\Pi_{11} + \frac{\partial f_2}{\partial z}\Pi_{21} + \frac{\partial f_3}{\partial z}\Pi_{31} & \frac{\partial f_1}{\partial z}\Pi_{12} + \frac{\partial f_2}{\partial z}\Pi_{22} + \frac{\partial f_3}{\partial z}\Pi_{32} & \frac{\partial f_1}{\partial z}\Pi_{13} + \frac{\partial f_2}{\partial z}\Pi_{23} + \frac{\partial f_3}{\partial z}\Pi_{33} \end{array} \right\}. \tag{15}$$

Let us examine, for example, the reverse action of a thin-walled spherical shell on the external space. According to (1), the expressions for the functions $f_1, f_2, f_3$ in a Cartesian coordinate system with origin at the center of the sphere can be represented in the form

$$f_1 = R^3 W_a \frac{x}{(x^2+y^2+z^2)^{3/2}}, \qquad f_2 = R^3 W_a \frac{y}{(x^2+y^2+z^2)^{3/2}},$$

$$f_3 = R^3 W_a \frac{z}{(x^2+y^2+z^2)^{3/2}}, \tag{16}$$

whence

$$F = \frac{R^3 W_a}{(x^2+y^2+z^2)^{5/2}} \left\{ \begin{array}{ccc} -2x^2+y^2+z^2 & -3xy & -3xz \\ -3xy & x^2-2y^2+z^2 & -3yz \\ -3xz & -3yz & x^2+y^2-2z^2 \end{array} \right\}. \tag{17}$$

The coefficient of reverse action on the external space is

$$W_a = \frac{\frac{1}{3}(K/2 - 2/K)\,\mathrm{sh}\,kd}{\mathrm{ch}\,kd + \frac{1}{3}(K+2/K)\,\mathrm{sh}\,kd} \simeq \frac{1}{3}\left(j\frac{Rd}{z_0^2} - \frac{d}{R}\right)\left(\text{for } \frac{Rd}{z_0^2} \ll 1;\ \frac{d}{R} \ll 1\right), \tag{18}$$

where $K = kR;\ k = \sqrt{j\omega\gamma\mu + 2/R}$, $z_0$ is the equivalent penetration depth.

We determine the components of the reverse-action field of the platform:

$$\dot{H}'_{ax} = \frac{R^3 W_a}{(x^2+y^2+z^2)^{5/2}} \left[(-2x^2+y^2+z^2)\Delta\dot{H}'_x - 3xy\Delta\dot{H}'_y - 3xz\Delta\dot{H}'_z\right],$$

$$\dot{H}'_{ay} = \frac{R^3 W_a}{(x^2+y^2+z^2)^{5/2}} \left[-3xy\Delta\dot{H}'_x + (x^2-2y^2+z^2)\Delta\dot{H}'_y - 3yz\Delta\dot{H}'_z\right],$$

$$\dot{H}'_{az} = \frac{R^3 W_a}{(x^2+y^2+z^2)^{5/2}} \left[-3xz\Delta\dot{H}'_x - 3yz\Delta\dot{H}'_y + (x^2+y^2-2z^2)\Delta\dot{H}'_z\right], \tag{19}$$

where $\Delta\dot{H}'_x, \Delta\dot{H}'_y, \Delta\dot{H}'_z$ must be computed using (9), (10). Suppose that $A = 0$, $\psi_x = \psi_y = \psi_z = 0$. For the external space, for example along the $z$-axis, on the basis of (9), (10), (18), (19) we have

$$\dot{H}'_{ax1} = \frac{1}{3}\sqrt{\left(\frac{Rd}{z_0^2}\right)^2 + \left(\frac{d}{R}\right)^2}\, T \sin I_0\, \Delta\alpha_{ym}\, \frac{R^3}{z^3} e^{-j\,\mathrm{arc\,tg}\,R^2/z_0^2},$$

$$\dot{H}'_{ax2} = \frac{1}{12}\sqrt{\left(\frac{Rd}{z_0^2}\right)^2 + \left(\frac{d}{R}\right)^2}\, T\cos I_0\,(\Delta\alpha_{ym}^2 + \Delta\alpha_{zm}^2)\,\frac{R^3}{z^3}e^{-j(\pi/2+\operatorname{arc\,tg} R^2/z_0^2)},$$

$$\dot{H}'_{ay1} = \frac{1}{3}\sqrt{\left(\frac{Rd}{z_0^2}\right)^2 + \left(\frac{d}{R}\right)^2}\, T(\sin I_0\,\Delta\alpha_{xm} - \cos I_0\,\Delta\alpha_{zm})\,\frac{R^3}{z^3}e^{j(\pi-\operatorname{arc\,tg} R^2/z_0^2)},$$

$$\dot{H}'_{ay2} = 0 \quad (A = 0), \tag{20}$$

$$\dot{H}'_{az1} = \frac{2}{3}\sqrt{\left(\frac{Rd}{z_0^2}\right)^2 + \left(\frac{d}{R}\right)^2}\, T\cos I_0\,\Delta\alpha_{ym}\,\frac{R^3}{z^3}e^{-j\operatorname{arc\,tg} R^2/z_0^2},$$

$$\dot{H}'_{az2} = \frac{1}{6}\sqrt{\left(\frac{Rd}{z_0^2}\right)^2 + \left(\frac{d}{R}\right)^2}\, T\sin I_0\,(\Delta\alpha_{xm}^2 + \Delta\alpha_{ym}^2)\,\frac{R^3}{z^3}e^{j(\pi/2-\operatorname{arc\,tg} R^2/z_0^2)}.$$

Let us quantitatively estimate the field magnitudes for the case of an aluminum shell ($\gamma = 38\cdot10^6\ \Omega^{-1}\mathrm{m}^{-1}$) under the conditions: $R = 0.25$ m; $d = 2\cdot10^{-3}$ m; $f = 0.4$ Hz ($z_0 = 0.125$ m); $\Delta\alpha_{xm} = \Delta\alpha_{ym} = \Delta\alpha_{zm} = 0.1$ rad.; $T = 5\cdot10^4$ gamma; $I_0 = 60^\circ$ ($Rd/z_0^2 = 32\cdot10^{-3}$; $d/R = 8\cdot10^{-3}$). Substituting these data into formulas (20), we have

$$\dot{H}'_{ax1} = 47.63\,\frac{R^3}{z^3}e^{-\operatorname{arc\,tg} R^2/z_0^2}\ \text{gamma}, \qquad \dot{H}'_{ax2} = 1.37\,\frac{R^3}{z^3}e^{-j(\pi/2+\operatorname{arc\,tg} R^2/z_0^2)}\ \text{gamma},$$

$$\dot{H}'_{ay1} = 20.13\,\frac{R^3}{z^3}e^{j(\pi-\operatorname{arc\,tg} R^2/z_0^2)}\ \text{gamma}, \qquad \dot{H}'_{ay2} = 0, \tag{21}$$

$$\dot{H}'_{az1} = 55\,\frac{R^3}{z^3}e^{-j\operatorname{arc\,tg} R^2/z_0^2}\ \text{gamma}, \qquad \dot{H}'_{az2} = 4.76\,\frac{R^3}{z^3}e^{j(\pi/2-\operatorname{arc\,tg} R^2/z_0^2)}\ \text{gamma}.$$

The corrections $\delta_x$, $\delta_y$, $\delta_z$, which take into account the inaccuracy of transformations (7) and (8), for each of the field components $\Delta H_x$, $\Delta H_y$, $\Delta H_z$ (or $H_x$, $H_y$, $H_z$) have, respectively, the magnitudes

$$\delta_x/H_x = \delta_y/H_y = \delta_z/H_z = (H - \sqrt{H^2 + \Delta})/H, \tag{22}$$

where

$$\Delta = \frac{1}{4}(\Delta\alpha_y^2 + \Delta\alpha_z^2)^2 H_x^2 + \frac{1}{4}(\Delta\alpha_x^2 + \Delta\alpha_z^2)^2 H_y^2 +$$

$$+\frac{1}{4}(\Delta\alpha_x^2 + \Delta\alpha_y^2)^2 H_z^2 + \left[(\Delta\alpha_x^2 - \Delta\alpha_y^2)\Delta\alpha_z - 2\Delta\alpha_x\Delta\alpha_y\right] H_{xH} y +$$

$$+ \left[(\Delta\alpha_z^2 - \Delta\alpha_x^2)\Delta\alpha_y - 2\Delta\alpha_x\Delta\alpha_z\right] H_{xH} z +$$

$$+ \left[(\Delta\alpha_y^2 - \Delta\alpha_z^2)\Delta\alpha_x - 2\Delta\alpha_y\Delta\alpha_z\right] H_{yH} z.$$

In a coordinate system associated with a fixed (for example stabilized) platform, the obtained result can be refined by means of the transformation inverse to (8),

$$\mathbf{H} = \mathbf{H}' - \vec{\Pi}_1 \times \mathbf{H}' + \Pi_2 \mathbf{H}'. \tag{23}$$

In some cases one may also use the transformation inverse to transformation (5), which has the form

$$\mathbf{H} = \Pi^{-1}\Delta\mathbf{H}', \tag{24}$$

where

$$\Pi^{-1} = \frac{1}{D(\Pi)} \left\{ \begin{matrix} \Pi_{1x}^2 + \frac{1}{2}(\Pi_{1y}^2 - \Pi_1^2) & \frac{1}{2}(\Pi_{1z}^2 - \Pi_1^2) & \Pi_{1x}\Pi_{1y} + \Pi_{1z}\frac{1}{2}(\Pi_{1z}^2 - \Pi_1^2) \\ \Pi_{1x}\Pi_{1y} - \Pi_{1z}\frac{1}{2}(\Pi_{1z}^2 - \Pi_1^2) & \Pi_{1y}^2 + \frac{1}{2}(\Pi_{1x}^2 - \Pi_1^2) & \frac{1}{2}(\Pi_{1z}^2 - \Pi_1^2) \\ \Pi_{1x}\Pi_{1z} + \Pi_{1y}\frac{1}{2}(\Pi_{1y}^2 - \Pi_1^2) & \Pi_{1y}\Pi_{1z} - \Pi_{1x}\frac{1}{2}(\Pi_{1x}^2 - \Pi_1^2) & \Pi_{1z}^2 + \frac{1}{2}(\Pi_{1x}^2 - \Pi_1^2)\frac{1}{2}(\Pi_{1y}^2 - \Pi_1^2) \end{matrix} \right\}. \tag{25}$$

Transformation (24) makes it possible to calculate the primary field from the interference caused by rotation.

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## CITED LITERATURE

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*Note: Figure translations are in progress. See original paper for figures.*