MATHEMATICS

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Abstract

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MATHEMATICS

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GENERALIZED REGULARLY MONOTONE FUNCTIONS AND A CRITERION FOR ABSOLUTE CONVERGENCE OF A QUASI-POWER SERIES

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Definition 1. A function $\varphi(t)$ on $(0, u]$, $u > 0$, belongs to the class of generalized regularly monotone functions with respect to the sequence of numbers

$$0 = \gamma_0 < \gamma_1 \leqslant \gamma_2 \leqslant \cdots, \tag{1}$$

briefly, to the class $R_\gamma(0, u]$, if on $(0, u]$ there exists a sequence of functions

$$\varphi_0(t) = \varphi(t), \quad \varphi_1(t) = \varphi'(t), \quad \varphi_{k+1}(t) = \left(\frac{\varphi_k(t)}{t^{\gamma_k - \gamma_{k-1} - 1}}\right)', \quad k = 1, 2, \ldots, \tag{2}$$

and the conditions

$$(-1)^k \varphi_k(t) \geqslant 0. \tag{3}$$

are satisfied.

Definition $1'$**.** A function $\varphi(t)$ belongs to the class of functions $R_{\gamma,\varkappa}(0, u]$ if it belongs to the class $R_\gamma(0, u]$ and, moreover, the lower bound of all numbers μ for which $\lim\limits_{t \to +0} t^\mu \varphi(t) = 0$ is equal to $\varkappa$.

Definition 2. A function $\varphi(t)$ belongs to the class of functions $T_\gamma(0, u]$ if it can be expanded into a series converging on $(0, u]$ to the function $\varphi(t)$,

$$\varphi(t) = \sum_{k=0}^{\infty} a_k \omega_k \left(\frac{t}{u}, \gamma\right) \tag{4}$$

(see ([1])). If the order of uniform convergence of the series (4) on $[0, u]$ is then equal to $\varkappa$, then the class $T_\gamma(0, u]$ will be denoted by $T_{\gamma,\varkappa}(0, u]$.

If, instead of ordinary convergence, absolute convergence of the series (4) on $(0,u]$ is allowed, then the corresponding classes of functions will be denoted by $AT_\gamma(0,u]$ and $AT_{\gamma,\varkappa}(0,u]$.

Definition 3. A function $\varphi(t)$ belongs to the class of functions $AC_{\gamma,\varkappa}(0,u]$, where $\varkappa \geqslant 0$ is an arbitrary number, the sequence of numbers $\{\gamma_\nu\}$ is defined in (1), if for $t \in (0,u]$ there exists a sequence of functions (2) and, moreover, the conditions are satisfied

$$\int_0^u |\varphi_{n+1}(t)|\, t^{\gamma_n+\varkappa_1}\, dt \leqslant C \prod_{\nu=1}^{n} (\varkappa' + \gamma_\nu), \quad n = 1, 2, \dots, \tag{5}$$

where $\varkappa_1$ and $\varkappa'$ ($\varkappa_1 > \varkappa' > \varkappa \geqslant 0$) are arbitrary numbers; C is a constant independent of n.

Theorem 1. Let there be given sequences of numbers $0 = \gamma_0 < \gamma_1 \leqslant \gamma_2 \leqslant \cdots$, $0 = \gamma'_0 < \gamma'_1 \leqslant \gamma'_2 \leqslant \cdots$, where $\gamma_\nu \leqslant \gamma'_\nu$, $\nu = 1, 2, \dots$,

and the corresponding classes of functions $R_\gamma(0,u]$, $R_{\gamma'}(0,u]$. Then

$$R_\gamma(0,u] \subset R_{\gamma'}(0,u].$$

The converse assertion, generally speaking, is false.

Theorem 2. Every function $\varphi(t) \in R_\gamma(0,u]$ also belongs to the class $T_\gamma(0,u]$, if and only if the sequence (1) satisfies the condition

$$\sum_{\nu=1}^{\infty} \frac{1}{\gamma_\nu} = \infty. \tag{6}$$

Theorem 3. In order that a function $\varphi(t) \in AT_\gamma(0,u]$, where the sequence $\{\gamma_\nu\}$ defined in (1) satisfies condition (6), it is necessary and sufficient that it be representable as the difference of two functions of the class $R_\gamma(0,u]$.

To clarify the interrelation of the classes of functions $AT_\gamma(0,u]$ and $R_\gamma(0,u]$, the following is proved:

Theorem 4. In order that $\varphi(t) \in AT_{\gamma,\chi}(0,u]$, $\chi \geq 0$, where the sequence $\{\gamma_\nu\}$ is defined in (1) and (6), it is necessary and sufficient that $\varphi(t)$ be representable as the difference of two functions $\psi(t)$ and $g(t)$, where $\psi(t) \in R_{\gamma,\mu}(0,u]$, $g(t) \in R_{\gamma,\mu'}(0,u]$, $\mu \leq \chi$, $\mu' \leq \chi$, and in at least one of the last inequalities the equality sign holds.

Theorem 5. In order that $\varphi(t) \in AT_{\gamma,\chi}(0,u]$, $\chi \geq 0$, where the sequence of numbers (1) satisfies condition (6), it is necessary and sufficient that $\varphi(t) \in AC_{\gamma,\chi}(0,u]$.

Theorem 6. For the quasianalyticity of the class of functions $AC_{\gamma\chi}(0,u]$ (see ([1])) it is necessary and sufficient that the sequence of numbers (1) satisfy condition (6).

From Theorem 6 and Theorem 2 of note ([2]) it follows:

Theorem 7. The totality of all quasianalytic classes $C_{\gamma,\chi}(0,u]$ (see ([2])) coincides with the totality of all quasianalytic classes of functions $AC_{\gamma,\chi}(0,u]$, whereas for the specific classes $C_{\gamma,\chi}(0,u]$ and $AC_{\gamma,\chi}(0,u]$ there is a strict inclusion

$$AC_{\gamma,\chi}(0,u] \subset C_{\gamma,\chi}(0,u].$$

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[6] G. V. Badalyan, DAN, **136**, No. 1 (1961). [2] G. V. Badalyan, DAN, **141**, No. 5 (1961).

Note: Figure translations are in progress. See original paper for figures.