Soviet-era science, translated into English

CYBERNETICS AND CONTROL THEORY

==================================

1962

SovietRxiv

View the original and related papers at https://sovietrxiv.org/items/ru-196201.32442

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

SovietRxiv

Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

G. M. ULANOV

OPTIMIZATION OF AUTOMATIC CONTROL SYSTEMS
AND THE THEORY OF $K(D)$-IMAGES

(Presented by Academician A. Yu. Ishlinskii, 16 I 1962)

The general theory of optimization of automatic control systems has as its subject the optimal synthesis of automatic control systems operating under continuously acting disturbances.

In the deterministic formulation of the problem ([1–3, 7, 8]), the criterion of optimality is the attainment of the highest accuracy of an automatic control system, measured by the error ε, equal to the difference between the desired W and the realized W^* value of the state of the system, $\varepsilon = W - W^*$. In statistical synthesis, the optimal system—found from the probabilistic characteristics of the control action and disturbances, and whose transfer function is Φ_{opt}—corresponds to the greatest accuracy only on the average for the whole aggregate of conditions ([4]).

The principal task of the theory of optimal systems consists in determining Φ_{opt}, which effects the approximation of one random function, by transformation of its system into another, as accurately as possible. The latter is evaluated by a loss function l, which in the simplest case is equal to ε, and in the general case represents some function $l(W, W^*)$ ([5, 6]). The criterion of optimality is usually the minimum of the mathematical expectation of the loss function $l(W, W^*)$ and, in particular, the minimum of the mean risk $M[l(W, W^*)] = \min$.

The main results concerning the construction of optimal systems in the deterministic formulation have been obtained by the theory of invariance; on their basis, automatic control systems have been constructed with error ε equal to zero or very small in the presence of disturbances whose measurement or use for control purposes is possible. The conditions of the theory of invariance of automatic control systems in the case when the disturbance links do not reduce to zero the numerator of the transfer function (and hence the corresponding transfer function) are expressed with the aid of the $K(D)$-image of functions introduced by V. S. Kulebakin:

$$K(D)f(t) = 0, \qquad K(D) \not\equiv 0, \qquad f(t) \not\equiv 0. \tag{1}$$

$K(D)$ and $f(t)$ are connected by the conditions of the operational $K(D)$-image of functions (1). In this case, for a stable system its transfer function must

SovietRxiv

either be identically the $K(D)$-image or have this operational $K(D)$-image as a factor.

In the statistical formulation of the optimization problem, the disturbance $f(t)$ is regarded as a certain random function with a known distribution law (moments of the function). The principal applied results in determining the transfer function of a control system in the case of its infinite memory, according to the criterion of minimum mean-square error, are due to Wiener. It is evident that in one case it is possible to establish exactly the correspondence of optimal systems in the statistical and deterministic formulations of the problem ([7, 8]). As the variance of $f(t)$ tends to zero, the optimal Wiener system and the optimal system determined by

conditions of invariance coincide, in the formulation of their problems, and, strictly speaking, should lead to one and the same systems. Let us show the commonality of the systems obtained in this case by Wiener and of invariant systems and, in particular, those satisfying the condition of the $K(D)$-image of V. S. Kulebakin. If the observation interval $f(t)$ is taken to be infinite and, thus, one is interested only in the forced "output" of the system, then the transfer function of the Wiener-optimal system is determined by the expression [(4–6)]:

$$\Phi_{\text{opt}}(j\omega) = \frac{1}{2\pi\psi(j\omega)} \int_0^\infty e^{-j\omega t}\, dt \int_{-\infty}^\infty \frac{S_{h\varphi}(\omega)}{\psi^*(j\omega)} e^{j\omega t}\, d\omega, \tag{2}$$

where $|\psi(j\omega)|^2 = S_f(\omega)$ is the spectral density of the external disturbance of the automatic-control system, and $S_{h\varphi}$ is the cross-spectral density of the desired process and the external disturbance.

In the indicated optimal system with transfer function $\Phi_{\text{opt}}(j\omega)$, the value of the mean-square error $\bar{\varepsilon}^2$ is determined [(6)] by the expression

$$\bar{\varepsilon}^2 = \frac{1}{2\pi} \int_{-\infty}^\infty \left\{ S_h(\omega) - |\Phi_{\text{opt}}(j\omega)|^2 S_f(\omega) \right\} d\omega, \tag{3}$$

where S_f is the spectral density of $f(t)$, and $S_h(\omega)$ is the spectral density of the desired signal.

In the regulation problems under consideration, when the stabilization condition is satisfied, $S_h(\omega)$ is identically equal to zero, since with complete filtering of the external disturbance $f(t)$ the desired output of the system must be identically equal to zero. The conditions for zero error $\bar{\varepsilon}^2_{\min} = 0$ in this case (according to Wiener) lead to the following requirement with respect to the optimal transfer function of the automatic-control system:

$$\varepsilon^2 = 0, \quad S_h(\omega) = 0,$$

SovietRxiv

$$|\Phi_{\text{opt}}(j\omega)|^2 S_f(\omega) = 0. \quad (4)$$

In the case where $\Phi_{\text{opt}} \neq 0$ and $S_f \neq 0$, fulfillment of condition (4) is possible when

$$\Phi_{\text{opt}}(D)\, f(t) = 0. \quad (5)$$

The indicated requirement corresponds to the condition of invariance of the Wiener-optimal system to the disturbance $f(t)$, expressed by the $K(D)$-image (1). In other words, the Wiener-optimal system Φ_{opt}, when the variance of $f(t)$ tends to zero, must be a $K(D)$-image of $f(t)$, $K(D) \div f(t)$, or contain it as a factor. The commonality of invariant systems and Wiener-optimal systems is established analogously in the case of more general control problems. Thus, the use of the $K(D)$-image is a powerful tool for the theory of optimization of automatic-control systems.

Received
25 XII 1961

CITED LITERATURE

1. V. S. Kulebakin, UMN, **6**, no. 5, 211 (1951); DAN, **68**, no. 5 (1949); **77**, no. 2 (1951).

2. B. N. Petrov, G. M. Ulanov, *Some Problems of Combined Automatic-Control Systems*, Proceedings of a Session of the Academy of Sciences of the USSR, no. 5, 1956.

3. A. Yu. Ishlinskii, Proceedings of the Conference on the Theory of Invariance and Its Applications, Academy of Sciences of the Ukrainian SSR, 1959.

4. N. Wiener, *Extrapolation, Interpolation and Smoothing of Stationary Time Series*, N. Y.—London, 1949.

5. V. S. Pugachev, *Theory of Random Functions*, 1957.

6. V. V. Solodovnikov, *Statistical Dynamics of Linear Automatic-Control Systems*, 1960.

7. A. G. Ivakhnenko, *Electrical Automatics*, pts. I, II, Publishing House of the Academy of Sciences of the Ukrainian SSR, 1957.

8. G. M. Ulanov, *Control by Disturbance*, 1960.

SovietRxiv

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.