Academician of the Academy of Sciences of the BSSR N. N. Sirota

1962

SovietRxiv

Abstract

Full Text

Physics

Academician of the Academy of Sciences of the BSSR N. N. Sirota

On the Temperature Dependence of the Electrical Conductivity of Solids

In works ($^{1-4}$) and others it has been convincingly shown that the main cause of the increase in the specific resistance of metals with increasing temperature is the scattering of current carriers by vibrations of ions in the crystalline lattice. Below we shall consider this question semi-quantitatively, relying on the results of Mott and Jones. In the subsequent derivations we shall not analyze the role of the frequency spectrum, remaining within the Debye approximation.

According to the approximate expression of the electron theory of metals, the specific electrical resistance is satisfactorily described by the expression

$$\rho = \frac{2m^*}{ne^2}\frac{1}{\tau}, \tag{1}$$

where m^* is the effective mass of the electron; e is the charge of the electron, n is the effective concentration of electrons—current carriers; τ is the mean free-flight time between collisions (the relaxation time).

The reciprocal of the relaxation time is proportional to the square of the amplitude of ion vibrations (^{4}). In contrast to the theory of Mott and Jones, we shall henceforth assume that the reciprocal relaxation time is directly proportional to the sum of the mean-square dynamic $\overline{u^2_{S,T}}$ and static $\overline{u^2_{S,C}}$ displacements of ions from the equilibrium position in the crystalline lattice. In this case, instead of expression (1) we obtain

$$\rho = \frac{2m^*}{ne^2}\left(\overline{u^2_{S,T}} + \overline{u^2_{S,C}}\right). \tag{2}$$

From expression (2) there follows directly Matthiessen–Fleming’ s rule

$$\rho = \rho_T + \rho_C, \tag{3}$$

which makes it possible to divide the specific electrical resistance of metals into two parts: a temperature-dependent part ρ_T and a part practically independent of temperature, ρ_C, determined by the presence of impurities and defects.

The concentration dependence of the mean-square static displacements caused by impurities, in the simplest case of a two-component regular solution, can be described by an expression of the type $\overline{u^2_{S,C}} \simeq KC_A^q(1-C_A)^p$, where the quantity K depends on the ratio of atomic volumes and on the elastic properties of the solvent. In the simplest case $q \simeq p \simeq 1-2$.

The mean-square dynamic displacements can be determined experimentally from the temperature variation of the intensities of X-ray scattering by the crystalline lattice of a solid.

The Debye–Waller theory of the temperature factor M_T of X-ray scattering by a crystalline lattice (^{5}) makes it possible to calculate the quan-

quantity of root-mean-square displacements as a function of temperature, if the characteristic temperature θ is known:

$$u^2_{S,T} = \frac{3\hbar^2}{Mk\theta}\left[\frac{\Phi\chi}{\chi} + \frac{1}{4}\right] = \frac{M_T}{8\pi^2}\frac{\lambda^2}{\sin^2\vartheta}, \tag{4}$$

where

$$\overline{\Phi}(\chi) = \frac{1}{\chi}\int_0^{\chi}\frac{x\,dx}{e^x-1}, \qquad \chi = \frac{\theta}{T},$$

M is the mass of the ion. From relations (2), (4) we obtain the expression

$$\rho_T = \frac{6m^*\hbar^2}{ne^2Mk\theta}\left[\left(\frac{T}{\theta}\right)^2\int_0^{\theta/T}\frac{x\,dx}{e^x-1} + \frac{1}{4}\right], \tag{5}$$

which describes the temperature dependence of the specific electrical resistivity of metals at constant electron concentration and effective mass. In a qualitative consideration of the question, we omit an analysis of the influence of transfer phenomena, etc. Their successive consideration leads to a temperature dependence $\rho(T)$ at low temperatures proportional to $(T/\theta)^5$ (4). However, for many substances and, in particular, semiconductors, it is possible to confine oneself to the approximation considered here.

Fig. 1 compares the experimentally determined temperature dependence of the specific electrical resistivity of several metals with that calculated by formula (5). If the concentration of current carriers depends on temperature, as for example in semiconductors, according to the exponential law $n = n_0e^{-\Delta E/2kT}$, then

$$\rho_T = \frac{6m^*\hbar^2}{n_0e^2Mk\theta}e^{\Delta E/2kT}\left[\left(\frac{T}{\theta}\right)^2\int_0^{\theta/T}\frac{x\,dx}{e^x-1} + \frac{1}{4}\right]. \tag{6}$$

Figure 2

Figure 1: Figure 2

In the case of mixed conductivity, caused by the motion of negative and positive current carriers, putting $m_e^*/n_{0e}e^2 = A_e$, $m_p^*/n_{0p}^+e^2 = A_p$, we obtain

$$\rho_T = \frac{6\hbar^2}{Mk\theta}\left[\left(\frac{T}{\theta}\right)^2 \int_0^{\theta/T} \frac{x\,dx}{e^x - 1} + \frac{1}{4}\right] \times$$

$$\times \left(A_e e^{\Delta E_e/2kT} + A_p e^{\Delta E_p/2kT}\right). \tag{7}$$

Fig. 1. Change in the specific resistivity of Al, Au, Pb, Ag as a function of temperature:
a—experimental values,
b—theoretical values

Thus, the temperature dependence of the electrical resistivity is determined by functions that include characteristic temperatures and activation energies, directly related to the lattice energy. Consequently, from this point of view, the temperature dependence of the electrical resistivity is to a large extent connected with the lattice energy of a solid.

In Fig. 2 the experimental and theoretical values of the specific impurity resistivity of silicon are compared as a function of temperature.

In a number of metals the conductivity is due both to the electrons of the metal, which are scattered by dynamic and static displacements, and to electrons directly bound to impurities, whose transfer into the conduction band requires an activation energy. (The valence-band electrons of the metal may also partly participate in the conductivity; their transfer into the conduction band requires an activation energy.)

Fig. 2. Change in specific resistivity as a function of temperature for two samples of germanium doped with arsenic: *a*—Debye' s experimental values (6), *b*—theoretical values.

On the basis of the above, the specific resistance of a metal with a concentration of impurity of the indicated type can be expressed by the relation

$$\rho = \rho c + \frac{6m^*\hbar^2}{n_0 e^2 kM\theta}\left[\frac{\Phi(\chi)}{(\chi)} + \frac{1}{4}\,\frac{B}{1 + \frac{n_{01}m_1^*}{n_0 m^*}e^{-\Delta E/2kT}}\right]. \tag{8}$$

In some cases the first term in the denominator of the last factor may be neglected, and then we obtain an expression analogous to (6). If the second term in

Figure 3

Figure 2: Figure 3

the denominator of the last factor is omitted, then formula (8) becomes formula (5).

Fig. 3. Change in specific resistivity as a function of temperature for two samples of gold and one sample of silver: *a*—experimental values (7), *b*—theoretical values.

In Fig. 3 the experimental data and the data calculated from expression (8) are compared for the temperature dependence of two samples of gold and one sample of silver; they show good agreement between theory and experiment.

Department of Solid-State Physics and Semiconductors
Academy of Sciences of the BSSR

Received
15 XI 1961

CITED LITERATURE

1. W. Wien, *Sitzungsber. Preuss. Akad. Wiss.*, **5**, 184 (1913).
2. F. Bloch, *Zs. Phys.*, **52**, 555 (1928); **53**, 216 (1929); **59**, 208 (1930).
3. E. Grüneisen, *Ann. d. Phys.*, **16**, 530 (1933).
4. N. F. Mott, T. Jones, *The Theory of the Properties of Metals and Alloys*, Oxford, 1936; Russian transl., 1958.
5. P. James, *Optical Principles of the Diffraction of X-rays*, IL, 1960.
6. *Collection: Germanium*, IL, 1955, p. 125.
7. *Collection: Low-Temperature Physics*, IL, 1959, p. 169.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.