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Abstract

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THE INFLUENCE OF NOISES ON THE SPECTRAL COMPOSITION AND ANGULAR DISTRIBUTION OF THE GENERATION OF A FINITE PLANE-PARALLEL LAYER

1. The radiation density at a point z inside a plane-parallel layer is determined by the formula ([1])

$$u = u_0(1-r)\,\frac{e^{-kz} + re^{-2kl+kz} - 2\left(\frac{\chi}{n}\right)^2 e^{-kl}\sqrt{r}\cos\left[\frac{4\pi\nu n}{c}(l-z) - \delta\right]}{(1-re^{-kl})^2 - 4re^{-kl}\sin^2\left(\frac{2\pi\nu nl}{c} - \delta\right)}, \quad (1)$$

obtained by an exact solution of Maxwell' s equations with allowance for the boundary conditions. Here u_0 is the density of the radiation flux incident from outside on the front boundary of the layer; r is the reflection coefficient; n and k are the refractive index and absorption coefficient of the material of the layer; $\chi = kc/4\pi\nu$; ν is the frequency; c is the speed of light; l is the thickness of the layer;

$$\delta = \operatorname{arctg}\frac{2n\chi}{n_{\text{cp}}^2 - n^2 - \chi^2};$$

n_{cp} is the refractive index of the medium*.

In media with a negative absorption coefficient the value of χ is always very small. In view of this, the pulsating term in the numerator of (1) may be neglected and one may put $\delta = 0$. Averaging (1) over the coordinate z, we obtain

$$\bar{u} = u_0(1-r)\frac{1-e^{-kl}}{kl}\,\frac{1+re^{-kl}}{(1-re^{-kl})^2 + 4re^{-kl}\sin^2\frac{2\pi\nu nl}{c}}. \quad (2)$$

If the density of the external radiation u_0 is equal to zero, then a finite value of u can occur only when the conditions

$$re^{-kl} = 1, \tag{3}$$

$$\frac{2\pi n l \nu}{c} = \pi s \quad (s \text{ is an integer}), \tag{4}$$

called generation conditions, are fulfilled. Similar results were obtained by directly solving Maxwell' s equations for $u_0 = 0$.

The correct value of all quantities characterizing the luminescence of a plane-parallel layer can be obtained only by taking into account the nonlinear dependence of the absorption coefficient on the radiation density inside the layer ([2])

$$k = \frac{k_0}{1 + \alpha u}. \tag{5}$$

Comparing (3) and (5), we obtain (see more detail in ([3]))

$$\bar{u}_{\text{gen}} = -\frac{k_0 l + \ln \frac{1}{r}}{\alpha \ln \frac{1}{r}}, \tag{6}$$

* The expression given for δ is valid in the absence of coatings, when r is determined by the Fresnel formulas. In the general case the expression changes insignificantly.

The spectrum of the layer' s intrinsic emission consists of several strictly monochromatic lines ($\Delta\nu = 0$) with frequencies determined by condition (4)*.

Formula (6) is valid not only in the wave-optics approximation, but also in the geometrical-optics approximation. Averaging result (2) over a slightly variable layer thickness ($\Delta l \gg \lambda$) leads to the expression

$$\bar{u} = u_0(1 - r)\,\frac{1 - e^{-kl}}{kl}\,\frac{1}{1 - re^{-kl}}, \tag{7}$$

which is also obtained by directly adding the densities of individual beams multiply reflected inside the layer from the boundary. From the derivation of (7) within geometrical optics ([3,4,6]) it follows that the radiation flux generated by a bounded plane-parallel layer propagates strictly normally to the surface of the plane-parallel layer. In all other directions φ there is no generation. Indeed, for all directions other than the normal, the number of summed beams N_φ remains finite, and the radiation density is determined by the expression

$$u_\varphi = u_0(1-r)\,\frac{1-e^{-kl'}}{kl'}\,\frac{1-(re^{-kl'})^{N_\varphi}}{1-re^{-kl'}}, \qquad l' = \frac{l}{\cos\varphi}, \tag{8}$$

and for $u_0 = 0$ is identically equal to zero. Generation can exist only when $N_\varphi \to \infty$ (i.e., when $\varphi = 0$).

2. In all real systems the density of the external radiation, considered below as noise, is never equal to zero. It includes not only Planck radiation, but also radiation arising from spontaneous emission. Inside a layer with a negative absorption coefficient it will be amplified many times and will produce luminescence of the same order of magnitude as (6) even when conditions (3) and (4) are not satisfied. The possibility of violating condition (4) leads to broadening of the spectral lines; the possibility of violating (3) leads to blurring of the beam.

In the subsequent calculations connected with the influence of noise, we shall use formulas (1), (2), (7), and (8), which are directly applicable only to the case when the radiation u_0 is incident on the layer from outside. However, when $u_0 \ll u_{\text{gen}}$, the results of the calculation will also be valid (up to numerical coefficients) in those cases when the occurrence of u is connected with processes taking place inside the layer. To describe processes occurring near the generation threshold ($u_0 \sim u_{\text{gen}}$), a special treatment is required.

3. Suppose that relation (3) is satisfied. If u_0 has a prescribed finite value, then it is not difficult to find the value of the frequency ν', close to ν_s, at which $\bar{u}$ in (2) will be equal to u_{gen}. Taking (3) into account, we obtain

$$\sin^2\left(\frac{2\pi nl\nu'}{c}\right) = \frac{u_0}{u_{\text{gen}}}\,\frac{(1-r)^2}{2r\ln\frac{1}{r}}. \tag{9}$$

Since $\nu' - \nu_s$ is very small, the spectral width is equal to

$$\Delta\nu = 2(\nu'-\nu_s) = \frac{c}{\pi nl}\,\frac{1-r}{\sqrt{2r\ln\frac{1}{r}}}\sqrt{\frac{u_0}{u_{\text{gen}}}} =$$

$$= \frac{c(1-r)}{\pi nl}\sqrt{\frac{\alpha u_0}{2r}\,\frac{1}{-k_0 l - \ln\frac{1}{r}}}\ \text{sec}^{-1}. \tag{10}$$

For an order-of-magnitude estimate one may put $r = 0.95,\ n = 1,\ l = 10$ cm. This gives

$$\Delta\nu = 10^8\sqrt{\frac{u_0}{u_{\text{gen}}}} = 10^7\sqrt{\frac{\alpha u_0}{-k_0 l - 0.05}}\ \text{sec}^{-1}. \tag{11}$$

* In paper (4) it is shown that the position of the spectral lines also depends on the value of k, determined by condition (3).

The ratio u_0/u_{gen} characterizes the ratio of noise to useful radiation. A decrease in the fraction of noise is accompanied by an increase in the degree of monochromaticity of the lines. If u_0/u_{gen} is very small, then the width of the emitted spectral lines is considerably less than the natural width.

In the present calculation, the value of $\Delta\nu$ has been determined for radiation emitted normal to the surface of the layer. It did not take into account the broadening of spectral lines associated with a violation of plane-parallelism and, for finite layers, that associated with diffraction phenomena. In the nonstationary regime, additional factors arise that lead to an increase in $\Delta\nu$.

4. The presence of noise leads to the appearance of luminescence not only in the direction normal to the surface of the layer, but also in other nearby directions. The calculation of the angular distribution is most simply carried out in the approximation of geometrical optics, without taking interference and diffraction phenomena into account. Suppose that an external isotropic flux with density u_0 is incident on a finite* plane-parallel layer. Further suppose that the thickness of the layer considerably exceeds the dimensions of its base, and that the side walls completely transmit all the radiation incident on them.** This makes it possible to consider only those beams that propagate inside the layer at small angles φ, and to neglect the angular dependence of the reflection coefficient r, the absorption coefficient k, and the path length traversed by the beam between the bases of the layer.

For $u_0 = 0$, the radiation density inside the generating layer is determined by formula (6). The presence of noise leads to a certain increase in the density and to a corresponding decrease in the value of the absorption coefficient. As a result, instead of (3) the relation

$$re^{-kl} < 1. \tag{3'}$$

will be valid.

The total radiation density inside the layer is obtained from (8) by integrating over all solid angles

$$\overline{u}' = \int_\Omega u_\varphi d\Omega = u_0'(1-r)\frac{1-e^{-kl}}{kl}\frac{1}{1-re^{-kl}}, \tag{12}$$

where

$$u_0' = u_0 \int_\Omega \left[1 - \left(re^{-kl}\right)^{N_\varphi}\right] d\Omega < \int_\Omega u_0 d\Omega$$

is a certain effective density of the external radiation.

The value of k in (12), and hence the true value $\overline{u}'$ established inside the layer in the presence of noise, can be obtained by numerically solving the system of equations (12) and (5). For small u_0' and, consequently, small deviations from (3), this gives***

$$k = \frac{1}{l}\ln\left(r + \frac{au_0'(1-r)^2}{k_0 l + au_0'(1-r) + \ln r}\right) =$$

$$= -\frac{1}{l}\ln\frac{1}{r} + \frac{(1-r)^2}{lr}\frac{u_0'}{u_{\text{gen}}\ln\frac{1}{r} + u_0'(1-r)}, \tag{13}$$

$$\overline{u}' = u_{\text{gen}} + u_0'\left(\frac{1-r}{\ln\frac{1}{r}}\right)^2\frac{1}{rau_{\text{gen}}}. \tag{14}$$

* An infinite plane-parallel layer possesses its own luminescence in different directions even for $u_0 = 0$ (7).

** In the presence of reflection at the side walls, the solid angle of the outgoing radiation increases.

*** Formula (13) gives only an estimate for k, since in fact the value of u_0' also depends on kl.

Since u_0' is small, $\bar{u}' \simeq u_{\text{gen}}$, and, consequently, the presence of spontaneous emission manifests itself not in an increase of the total generation power, but in its redistribution over angles: the appearance of luminescence in directions different from the normal one, and a simultaneous decrease of the generation intensity in the direction perpendicular to the layer.

The distribution of fluxes inside the layer over angles φ to the normal is determined by formula (8). The number N_φ, i.e., the maximum number of reflections at the surfaces of the layer before the flux exits through the side faces, is determined by the geometry of the generator. In the case of a cylinder it is approximately equal to

$$N_\varphi \simeq \frac{R}{l\,\text{tg}\,\varphi} \simeq \frac{R}{l\varphi}, \tag{15}$$

where R is the radius of the base. It follows from this that

$$\frac{\bar{u}_\varphi}{\bar{u}'} = \frac{1-\left(re^{-kl}\right)^{R/l\varphi}}{\int_\Omega\left[1-\left(re^{-kl}\right)^{R/l\varphi}\right]d\Omega}. \tag{16}$$

We define the width of the angular distribution as the doubled value of φ at which u_φ is two times smaller than its maximum value (in the normal direction, for $N_\varphi \to \infty$). This gives

$$\Delta\varphi = \frac{2R}{l \ln 2}\left(\ln\frac{1}{r} + kl\right).$$

Substituting the value of k from (13), we obtain an estimate for $\Delta\varphi$, expressed through u_0' and u_{gen}:

$$\Delta\varphi \simeq \frac{2R}{l \ln 2}\,\frac{(1-r)^2}{r}\,\frac{u_0'}{u_{\text{gen}} \ln\frac{1}{r}}. \tag{17}$$

As with the spectral width of the radiation (11), the value of $\Delta\varphi$ decreases as the noises decrease or the generation power increases. For $u_0' = 0$ the generation is strictly directed. For $R/l = 0.1$, $r = 0.95$, the value of $\Delta\varphi$, expressed in seconds, is equal to $\sim 10^3 u_0'/u_{\text{gen}}$. Already at $u_0'/u_{\text{gen}} \sim 10^{-3}$ the angular width of the beam amounts to several seconds. The greater the thickness of the layer and the smaller the dimensions of its base, the smaller $\Delta\varphi$.

5. The spectral and angular broadening of the generating beam caused by the action of noises are closely related to each other. From formulas (17) and (10) it follows that

$$\frac{(\Delta\nu)^2}{\Delta\varphi} = \left(\frac{c}{2\pi n}\right)^2 \frac{\ln 2}{Rl}. \tag{18}$$

This relation depends neither on the generation power, nor on the magnitude of the noises, nor even on the reflection coefficient of the layer surface.

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CITED LITERATURE

1. A. P. Kapalyuk, B. I. Stepanov, *Izv. AN SSSR, ser. fiz.-tekhn.*, vol. 4, 132 (1961).

2. B. I. Stepanov, *Dokl. AN BSSR*, 5, 489 (1961).

3. B. I. Stepanov, A. M. Samson, *DAN*, 142, 1282 (1962); *Izv. AN BSSR, ser. fiz.-tekhn.*, vol. 4, 45 (1961).

4. A. M. Samson, *Dokl. AN BSSR*, 6, 151 (1962).

5. B. I. Stepanov, *Dokl. AN BSSR*, 5, 541 (1961).

6. B. I. Stepanov, *Optika i spektroskopiya*, 12, 443 (1962).

7. A. M. Goncharenko, B. A. Sotskii, *Dokl. AN BSSR*, 6, no. 5 (1962).

Note: Figure translations are in progress. See original paper for figures.