GEOPHYSICS

1962

SovietRxiv

Abstract

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GEOPHYSICS

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ANALYSIS OF THE MONSOON FIELD OVER THE SEA WITH A COMPLEX SHAPE OF THE COASTLINE

(Presented by Academician V. V. Shuleikin, 12 IV 1962)

In his theory of monsoons ((*1*), pp. 460–489), V. V. Shuleikin arrives at a differential equation for the temperature anomaly τ of the atmosphere over the sea during the winter season:

$$\nabla^2\tau = \varkappa^2\tau - b^2. \tag{1}$$

A two-dimensional propagation of heat in a thin active layer of the atmosphere is considered. The quantities $\varkappa^2, b^2$ above the sea surface may, in a first approximation, be regarded as constant. The variability of $\varkappa^2$ over land apparently explains the empirical fact—the constancy of τ along the seacoast. To estimate the influence of the shape of the coastline on the intensity of the monsoon field, V. V. Shuleikin ((*1*), pp. 474–489), and then Ya. I. Sekerzh-Zenkovich ((*2*)), obtained approximate and exact solutions of equation (1) for certain simple curves, including the ellipse and the parabola. In order to clarify the character of the monsoon field over the sea for all the complexity of a real coastline, experimental investigations of the field on models were undertaken ((*3*, *4*)). The purpose of the present article is to show that in many cases of a complex coastline shape there exist ways of analytically solving the problem.

As boundary conditions we take $\tau_{\text{shore}} = c$ along a prescribed graphical contour.

Let a particular solution $\tau(x, y)$ of equation (1) be known such that the equation $\tau(x, y) = c$ defines a closed contour L in the plane. Obviously, the function $\tau(x, y)$ is in this case a solution of the problem for the contour L, and moreover is unique, if $\tau(x, y)$ satisfies the known requirements. Now let $\tau_1(x, y)$ be a solution of the homogeneous equation $\nabla^2\tau_1 = \varkappa^2\tau_1$. Since equation (1) is linear, the function $\psi = \tau + \alpha\tau_1$ will also be a solution of (1), but with the boundary-value problem for some other contour L_1, whose equation is $\tau(x, y) + \alpha\tau_1(x, y) = c$. We may regard the operation of adding the function τ_1 to the function τ as a deformation of the contour L into the contour L_1. By successively carrying out several predetermined deformations, one can obtain, first, the equation of the approximating contour

$$\tau(x, y) + \sum a_n \tau_n(x, y) = c,$$

and, second, an analytic solution of equation (1) in finite form.

A particular solution of the inhomogeneous equation (1) is easily obtained by knowing a particular solution of the homogeneous equation, by constructing the function $\tau = \tau_1 + b^2/\varkappa^2$. Indeed:

$$\nabla^2 \tau = \nabla^2 \tau_1 = \varkappa^2 \tau_1 = \varkappa^2(\tau - b^2/\varkappa^2) = \varkappa^2 \tau - b^2.$$

Putting $\tau_1 = X(x)Y(y)$, we find a solution of the homogeneous equation

$$X'' = kX, \qquad Y'' = lY, \qquad k + l = \varkappa^2.$$

Consider two classes of solutions:

1) $k = -m^2,\ l = n^2,\ n = \pm\sqrt{m^2 + \varkappa^2},\ X = A \cos mx + B \sin mx;$

$$Y = A_1 \operatorname{ch} ny + B_1 \operatorname{sh} ny, \quad \tau_1 = (A \cos mx + B \sin mx)(A_1 \operatorname{ch} ny + B_1 \operatorname{sh} ny);$$

2) $k = m^2,\ l = n^2,\ n = \pm\sqrt{\varkappa^2 - m^2},\ X = A \operatorname{ch} mx + B \operatorname{sh} mx,$

$$Y = A_1 \operatorname{ch} ny + B_1 \operatorname{sh} ny, \quad \tau_1 = (A \operatorname{ch} mx + B \operatorname{sh} mx)(A_1 \operatorname{ch} ny + B_1 \operatorname{sh} ny).$$

We shall here carry out an investigation of the winter monsoon field over the sea using the coastline of the Black Sea as an example, which will also serve as an illustration of the application of the general method. For definiteness and convenience in constructing the solution, let us take $\tau_{\text{shore}} = 0,\ \varkappa^2 = b^2 = 1$. As the first particular solution of (1), we take the function $\tau = 1 - \alpha \operatorname{ch} x - \beta \operatorname{ch} y$, composed of solutions of the second class. The equation

$$\alpha \operatorname{ch} x + \beta \operatorname{ch} y = 1 \tag{2}$$

defines in the plane an oval closed curve, which, when $\alpha = \beta$, becomes a circle slightly compressed at the points of intersection with the coordinate axes. To preserve the proportion of the smallest and largest distances between the shores of the Black Sea by latitude and longitude, we set the axes of the oval (2) equal to 3 and 0.75 units of the drawing scale. Substituting into equation (2) the coordinate values (0; 0.75) and (3; 0), we find the values of the coefficients α and β.

The equation $0.025\,\mathrm{ch}\,x + 0.75\,\mathrm{ch}\,y = 1$ determines the oval shown in Fig. 1a, and the function $\tau = 1 - 0.025\,\mathrm{ch}\,x - 0.75\,\mathrm{ch}\,y$ gives a solution of equation (1) for the domain bounded by the oval.

Keeping the magnitudes of the axes fixed as far as possible, we deform the curve by moving its points with abscissas $0 < x < 3$ away from the axis of abscissas. A function of the type $\gamma\,\mathrm{ch}\sqrt{1+m^2}\,y\cos mx$ from the first class of solutions for τ_1 can provide such a deformation if we put $m = 1.57$. Transferring the abscissa of the point of greatest deformation, $x = 2$, onto the map of the sea while preserving scale, we find that the ordinate at this point on the drawing scale should be equal to 1. This makes it possible to determine the value of the coefficient γ, by substituting the coordinates $(2, 1)$ into the equation of the deformed contour. The equation

$$0.025\,\mathrm{ch}\,x + 0.75\,\mathrm{ch}\,y + 0.077\,\mathrm{ch}\,1.865y\cos 1.57x = 1 \tag{3}$$

defines an oval with a "waist" (Fig. 1b), while the solution

$$\tau = 1 - 0.025\,\mathrm{ch}\,x - 0.75\,\mathrm{ch}\,y - 0.077\,\mathrm{ch}\,1.865y\cos 1.57x$$

for the domain has singular points of saddle type at the origin and centers $(\pm 1.65;\ 0)$, around which the isonomals swirl.

For a further deformation of the contour (3), we take a function that is odd in y and even in x. In the interval $1 < x < 2$, points with positive ordinates must be moved away from the axis of abscissas, while in the interval $2 < x < 3$, on the contrary, they must be brought closer, in order to form the rectilinear parts of the eastern and western shores. The lengths of the axes and the distance between the points $(2;\ \pm 1)$ should be left unchanged. Such a deformation can be provided by the function $-\delta\,\mathrm{sh}\,1.275y\cos 0.79x$. Taking the value of y near 0.5 at $x = 2.5$, so that this point, together with the points $(3;\ 0)$, $(2;\ 1)$, lies on a straight line, we determine the value of the coefficient $\delta = 0.313$. The equation of the contour after deformation and the solution for τ will have the form:

$$0.025\,\mathrm{ch}\,x + 0.75\,\mathrm{ch}\,y + 0.077\,\mathrm{ch}\,1.865y\cos 1.57x -$$

$$- 0.313\,\mathrm{sh}\,1.275y\cos 0.79x = 1,$$

$$\tau = 1 - 0.025\,\mathrm{ch}\,x - 0.75\,\mathrm{ch}\,y - 0.077\,\mathrm{ch}\,1.865y\cos 1.57x +$$

$$+ 0.313\,\mathrm{sh}\,1.275y\cos 0.79x.$$

Figure 1b shows the form of this contour. To smooth the southwestern and southeastern corners, we introduce one more function $ze^{-2.8y}\cos 2.62x$, which gives the maximum deformation of the curve for values of x near 2.5 and a negative value of y, and also belongs to the first class of solutions for τ_1. The final equation of the approximating contour and the solution for τ will be written as follows:

$$0.025\ \mathrm{ch}\, x + 0.75\ \mathrm{ch}\, y + 0.077\ \mathrm{ch}\, 1.865y \cos 1.57x -$$

$$-0.313\ \mathrm{sh}\, 1.275y \cos 0.79x + 3.77 \cdot 10^{-3} e^{-2.8y} \cos 2.62x = 1,$$

$$\tau = 1 - 0.025\ \mathrm{ch}\, x - 0.75\ \mathrm{ch}\, y - 0.077\ \mathrm{ch}\, 1.865y \cos 1.57x +$$

$$+0.313\ \mathrm{sh}\, 1.275y \cos 0.79x - 3.77 \cdot 10^{-3} e^{-2.8y} \cos 2.62x.$$

Figure 1c gives the appearance of the contour and of the theoretical isanomals. Study of the solution obtained shows that the temperature field has a special point of saddle type $(0; 0.45)$ and centers $(1.25, 0.4)$, which is due, as in (3), to the presence in the contour of a "waist." Just as in region (3), the temperature at the centers has a maximum value. The temperature difference on the contour and at the center is 0.23.

Let us find the gradient of the temperature field:

$$|\operatorname{grad} \tau| = \sqrt{(\tau_x')^2 + (\tau_y')^2},$$

$$\tau_x' = -0.025\ \mathrm{sh}\, x + 0.121\ \mathrm{ch}\, 1.865y \sin 1.57x -$$

$$0.247\ \mathrm{sh}\, 1.275y \sin 0.79x - 0.99 \cdot 10^{-2} e^{-2.8y} \sin 2.62x,$$

$$\tau_y' = -0.75\ \mathrm{sh}\, y - 0.143\ \mathrm{sh}\, 1.865y \cos 1.57x +$$

$$+0.40\ \mathrm{ch}\, 1.275y \cos 0.79x +$$

$$+1.05 \cdot 10^{-2} e^{-2.8y} \cos 2.62x.$$

Study of the expression for the gradient reveals the important fact that high values of it exist not only at capes, but also on rectilinear sections of the coastline,

Fig. 1

Figure 1: Fig. 1

especially near the points $(\pm 2; 1)$, i.e., somewhat south of Novorossiisk and near the mouth of the Danube.

Table 1 gives numerical values of the gradients for characteristic points.

Fig. 1

Table 1

x	0	± 0.8	± 1.7	± 2	± 2.5	± 3	± 2	0
y	1.1	1.3	1.25	1	0.45	-0.9	-1	-0.25
$\lvert \operatorname{grad} \tau \rvert$	0.69	0.62	0.48	0.68	0.67	0.35	0.59	0.71

It is evident from the table that the intensity of the field along the northwestern and northeastern coasts is of the same order as at the capes. The climatic features of these coasts are precisely characterized by stormy weather. The Novorossiisk bora, at any rate the monsoon-type bora, is apparently connected with an intensification of the monsoon field here. It is also known that catastrophic storms develop near the western coast of the sea.

Let us now estimate the influence of the Odessa Gulf and the Sea of Azov. When the ice cover ends on the Sea of Azov and the Odessa Gulf becomes chilled or is also covered over a considerable part by ice, this influence is absent or insignificant. On the contrary, in deep autumn, when the mainland has already become chilled, these waters still warm the air. To estimate the influence of this warming on the distribution of temperature gradients over the Black Sea, we intro-

the deforming function $\zeta e^{3.3y} \cos 3.14x$, which has the maximum deformation in the upper corners of the contour. At $\zeta = 1.365 \cdot 10^{-3}$ the contour breaks, forming straits in the upper corners (Fig. 1d). After the rupture the branches of the contour go off to infinity, bounding strips that at first widen, as is seen in the drawing, and then narrow slightly. Using deforming functions of the type $\eta e^{\sqrt{1+m^2}\, y} \cos mx$ with a higher value of m, it would be easy to close the branches of the contour; however, there is no need for this in studying the field over the Black Sea, where these functions with coefficients $|\eta| \ll \zeta$ and small ordinates practically disappear.

By studying the solution, one can detect a general increase of the gradients along the northern coast of the sea and a displacement of the coastal maxima near straight sections of the coast toward the north.

Thus, the well-known fact of the windiness of the climate of the northern coast can be explained from the conclusions of monsoon theory as the result of the action of a heat engine only of the second kind under conditions of a specific form of the boundary between the heater and the refrigerator. The November maximum of the action of the Novorossiisk bora can likewise be explained by the influence during this period of the Sea of Azov.

It remains for us to make a remark about the influence on the solution of the values χ^2, b^2. The unit of our scale contains about 200 km. Hence the adopted value of χ is $5 \cdot 10^{-8}$ cm^{-1}, $\chi^2 = 25 \cdot 10^{-16}$ cm^{-2}; V. V. Shuleikin in his works gives a value for χ^2 two orders of magnitude smaller (1). In accordance with this, we obtained and investigated the solution also for $\chi^2 = 0$, when equation (1) is transformed into the Poisson equation. The quantity b^2 was taken by us to be limited. The value of b^2 itself does not affect the distribution of gradients, since it does not depend on the coordinates. The result of investigating the solution shows that the general character of the field is preserved; in particular, the same regularities in the distribution of the coastal maxima of the gradient are observed. The difference of the solution of the Poisson equation consists in a slower decrease of the quantity $\text{grad}\,\tau$ when moving from the coast toward the central parts of the sea. We may conclude that the main features of the monsoon field over the Black Sea are preserved when χ^2 and b^2 vary within wide limits and are determined only by the form of the sea coastline.

In conclusion the author expresses gratitude to Acad. V. V. Shuleikin for valuable advice.

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Received
10 IV 1962

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Note: Figure translations are in progress. See original paper for figures.