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GEOPHYSICS

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Abstract

Full Text

GEOPHYSICS

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ON THE INFLUENCE OF A MAGNETIC FIELD ON THE INCREASE OF THE AMPLITUDE OF ACOUSTIC WAVES IN A MEDIUM WITH DECREASING DENSITY

(Presented by Academician M. A. Leontovich on December 23, 1961)

1. It is known that, when acoustic waves propagate in a medium of variable density, their velocity increases as the density decreases. This result is easily obtained in the linear approximation; however, in the case of a density falling to zero it remains valid only qualitatively, since the velocity tends to infinity and the formation of shock waves must be taken into account. The corresponding treatment ^(1,2) shows that, even after the formation of a shock wave, the acceleration continues up to distances of the order of the mean free path from the region of zero density. In the presence of a magnetic field and high conductivity, acceleration may continue also to larger amplitudes, since in this case the equations are analogous to purely gas-dynamic ones, and the characteristic length determining the limits of applicability of hydrodynamics is not the mean free path, but the Larmor radius of the ions ⁽³⁾. This conclusion is valid in the case when the magnetic pressure in the unperturbed regions falls to zero in the same way as the density.

If, however, the magnetic pressure falls more slowly, it may turn out that the presence of the field, on the contrary, prevents acceleration to large amplitudes, since as the density decreases the Alfvén velocity increases and, consequently, so does the rate at which energy is carried away. Owing to this circumstance, the cause of acceleration disappears—the transfer of finite energy to an infinitely decreasing mass of gas.

2. The limits of acceleration in the presence of a magnetic field can be estimated within the framework of the linear approximation. As an example, consider the case of an isothermal atmosphere in a uniform gravitational field \mathbf{g} . If the z -axis is directed along the normal to the boundary plane, the equilibrium distribution (Boltzmann law) is written in the form

$$\rho = \rho_0 e^{-z/z_0}, \quad p = p_0 e^{-z/z_0}, \quad z_0 = \frac{kT}{mg} \quad (1)$$

(here ρ is the density, p the pressure, k Boltzmann's constant, T the absolute temperature, and m the mass of a molecule). We shall assume that the magnetic field H_0 is uniform and perpendicular to \mathbf{g} . Suppose that the conductivity σ is sufficiently large, so that during the oscillations the condition of frozen-in field is satisfied, and that the dissipation associated with thermal conductivity and viscosity is also insignificant.

Consider small oscillations of the atmosphere along the z -axis with frequencies $\omega \ll eH_0/mc$ (e is the ion charge, c the speed of light). Under the assumptions made, the problem in all space is described by the equations of magnetohydrodynamics (to within the insignificant change of the adiabatic index γ from $5/3$ in the dense layers to 2 in the rarefied layers of the atmosphere).

For monochromatic oscillations the equations take the form

$$i\omega v = -\frac{1}{\rho} \frac{dp'}{dz} - g \frac{\rho'}{\rho} - \frac{1}{4\pi\rho} H_0 \frac{dH'}{dz}; \quad i\omega H' = -H_0 \frac{dv}{dz}; \quad (2)$$

$$i\omega\rho' + \frac{d}{dz} v\rho = 0; \quad i\omega \left(\frac{p'}{p} - \gamma \frac{\rho'}{\rho} \right) + v \frac{d}{dz} \ln p\rho^{-\gamma} = 0.$$

Here v, p', ρ', H' are perturbations of the velocity, pressure, density, and magnetic field; the last equation expresses the adiabatic nature of the motion. Eliminating all variables except v , with account of (1), we obtain

$$(\chi^2 + e^{-\xi}) \frac{d^2 v}{d\xi^2} - e^{-\xi} \frac{dv}{d\xi} + e^{-\xi} \lambda^2 v = 0, \quad (3)$$

where

$$\chi^2 = \frac{H_0^2}{4\pi\gamma p_0}, \quad \xi = \frac{z}{z_0}, \quad \lambda^2 = \frac{\omega^2 z_0}{\gamma g}. \quad (4)$$

At $\xi = 0$, $v = v_0$, while at infinity there must be no sources of oscillations.

In the absence of a magnetic field ($\chi^2 = 0$) we have:

$$v = v_0 \exp \left[\frac{1}{2} \xi \left(1 - i\sqrt{4\lambda^2 - 1} \right) \right]. \quad (5)$$

This solution formally corresponds to cumulative acceleration ($v \rightarrow \infty$ as $\xi \rightarrow \infty$). The quantity ρv^2 , however, remains bounded. (Of course, this result should be regarded only as purely qualitative, since for

$$v \gg \sqrt{\frac{kT}{m}} \gamma$$

the applicability of the linear approximation is violated, and as $\rho \rightarrow 0$ the viscosity increases without bound.)

However, for arbitrarily small χ different from zero, the behavior of the solution at infinity changes sharply.

Putting

$$x = 1 + \frac{1}{\chi^2} e^{-\xi}, \quad (6)$$

we reduce (3) to the hypergeometric equation:

$$x(1-x)v'' + (1-2x)v' - \lambda^2 v = 0 \quad (7)$$

with parameters satisfying the relations

$$\Gamma = 1, \quad \alpha + \beta = 1, \quad \alpha\beta = \lambda^2 \quad (8)$$

(to avoid confusion with the adiabatic exponent, we have denoted the third parameter by Γ , in contrast to the customary γ).

In the case $\lambda > 1/2$, α is complex and $\beta = \alpha^*$. Since the solution is sought in the region $x > 1$, it has the form⁴

$$v_1 = \left(\frac{1}{x}\right)^\alpha F\left(\alpha, \alpha, 2\alpha, \frac{1}{x}\right); \quad v_2 = v_1^* \quad (9)$$

(F is the hypergeometric function).

As $x \rightarrow 1$ ($\xi \rightarrow \infty$), both solutions diverge logarithmically. Let

$$v_1 = \theta \ln(x-1) + \operatorname{Re} g.$$

The linear combination of v_1 and v_2

$$v_3 = \operatorname{Im} \left\{ \theta^* \left(\frac{1}{x}\right)^\alpha F\left(\alpha, \alpha, 2\alpha, \frac{1}{x}\right) \right\} \quad (10)$$

remains finite also as $x \rightarrow 1$. As a second solution (in addition to v_3) one may take

$$v_4 = \operatorname{Re} \left\{ \theta^* \left(\frac{1}{x}\right)^\alpha F\left(\alpha, \alpha, 2\alpha, \frac{1}{x}\right) \right\}. \quad (10')$$

The solution obtained is meaningful for $z \ll c/\omega$, since we have neglected the displacement current. At large distances the solution becomes an electromagnetic wave. As a condition at infinity one may impose the requirement that for $z_0 \ll z \ll c/\omega$ the solution pass into an electromagnetic wave traveling in the direction $z \rightarrow \infty$. The particle velocity in the wave is cE/H_0 . But the electric field E in the traveling wave near $z = 0$ is

$$E_0 e^{i\omega(t-z/c)} \simeq E_0 e^{i\omega t} (1 - i\omega z/c),$$

i.e., accurate to terms $\sim \omega z_0/c$, the velocity is constant. If, however, the source of oscillations is located at $z \rightarrow \infty$, then the wave

is a standing one and, since by virtue of the ideal conductivity of the atmosphere, $E|_{z=0} = 0$, in the region $z_0 \ll z \ll c\omega$ we have $E \simeq E_0 e^{i\omega t} \omega z/c$.

It is easy to show that for $z \gg z_0$, $v_3 = \text{const}$, $v_4 \sim z$. Therefore, in the absence of sources at infinity, we must take the solution v_3 . Thus:

$$v = v_0 \frac{\text{Im}\{e^{-i\varphi}(1/x)^\alpha F(\alpha, \alpha, 2\alpha, 1/x)\}}{\text{Im}\left\{e^{i\varphi} \left(\frac{\chi^2}{1+\chi^2}\right)^\alpha F\left(\alpha, \alpha, 2\alpha, \frac{\chi^2}{1+\chi^2}\right)\right\}}, \quad (11)$$

where $\varphi = \arg \theta$.

For $\chi \ll 1$ and $x \ll 1$ (i.e., for $z \ll 2z_0 |\ln \chi|$)

$$v \simeq v_0 e^{\xi/2} \frac{\sin(\xi s + 2s \ln \chi - \varphi)}{\sin(2s \ln \chi - \varphi)}, \quad (12)$$

i.e., we have a solution of acoustic type with increasing velocity (here $s = \frac{1}{2} \text{Im } \alpha$). However, for $\xi \gg 2|\ln \chi|$ the acceleration ceases.

Let us note one more important difference between magnetohydrodynamic oscillations and purely acoustic ones: for

$$\omega \rightarrow \omega_n \simeq \frac{g}{2} \sqrt{\frac{\gamma m}{kT}} \sqrt{1 + \left(\frac{n\pi + \varphi_n}{\ln \chi}\right)^2} \quad (13)$$

resonance occurs. The calculation of the phase factor φ_n is difficult; however, it can be shown that $\varphi_n \rightarrow 0$ for small n , while for large n the role of φ_n is insignificant, since $|\varphi_n| \ll \pi$. Determination of the resonant frequencies by means of the asymptotic formula (12) is legitimate for $n \lesssim \frac{1}{\chi |\ln \chi|} \gg 1$. The presence of resonances is connected with the reflection of magnetohydrodynamic waves from regions with increasing Alfvén velocity.

From the physics of the problem (the unbounded increase of the rate of magnetohydrodynamic removal of energy as the density tends to zero) it is clear that these conclusions retain qualitative validity also in the case of other laws of density decrease, and also in the case of a field not perpendicular to \mathbf{g} .

3. The results obtained may be of interest for certain astrophysical questions. Thus, for example, it is known that chromospheric flares on the Sun occur mainly near neutral points of the magnetic field ⁵. If chromospheric flares are connected with the emergence at the surface of shock waves excited in the outer convective zone of the Sun, then the indicated regularity, in light of what has been set forth above, may be interpreted as a consequence of the cessation of wave acceleration in a medium with decreasing density in the presence of a magnetic field. In this case the greatest acceleration will occur precisely near neutral points, where the magnetic field is minimal.

In conclusion I take this opportunity to express my deep gratitude to Academician M. A. Leontovich for a number of important comments.

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Note: Figure translations are in progress. See original paper for figures.

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