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Abstract

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CONSTRUCTION OF TESTS FOR BLOCK DIAGRAMS

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This note considers the question of constructing tests for contact circuits. In doing so, we use the basic definitions and results of the article ([1]). The general algorithm for constructing minimal tests, given in ([1]), turns out to be extremely cumbersome even for circuits with a small number of contacts. Therefore it is natural to consider separate classes of contact circuits for which the construction of minimal or nearly minimal tests is simplified owing to the special features of the structure of the circuit. One such class is that of block diagrams.

Definition. An arbitrary l-terminal network is called a **block** if its l terminals are divided into two disjoint subsets; the l' terminals of the first subset are called inputs, and the l'' of the second are called outputs ($l' + l'' = l$).

Consider a sequence of blocks

$$B_1,\ B_2, \dots,\ B_n, \dots, \tag{1}$$

satisfying the following condition: the number of outputs of the block B_i is equal to the number of inputs of the block B_{i+1} ($i \geq 1$). To this sequence we associate the sequence of circuits

$$S_1,\ S_2, \dots,\ S_n, \dots \tag{2}$$

The circuit S^i is constructed as follows: the blocks $B_1, B_2, \dots, B_i$ are connected in series, i.e., the outputs of the preceding block are identified with the inputs of the following one; the inputs of the first block (the outputs of the last) are connected through a contact with the left (right) terminal of the circuit.

Definition. S_n is called a **block diagram**. In what follows, bounded block diagrams are studied.

Definition. A block diagram is called **bounded block** if the sequence (1) satisfies the following conditions: a) the number of contacts in each block does not exceed some constant K, independent of n; b) different blocks depend on different variables.

Fig. 1

Figure 1: Fig. 1

The number of contacts in such a circuit does not exceed $Kn + K_0$ (K_0 is the number of contacts incident to one of the terminals). Consequently, the length $t_{\min}$ of a unit minimal test for opening or closing lies within the limits ([1])

$$\log_2(Kn + K_0) \leqslant t_{\min} \leqslant Kn + K_0. \tag{3}$$

We shall study the possibility of constructing tests whose length approaches the lower bound (3).

Unit opening tests. Let us formulate a condition under which, for a bounded block diagram, one can construct a unit opening test of length not exceeding $c \log_2 n$.

Number the inputs and contacts in the blocks. The inputs of the m-th block ($1 \leq m \leq n$) will be denoted by $a_{m1} \dots a_{ml_m}$, and the contacts by the letters $p_1, p_2, \dots, q_1, q_2, \dots$. The left terminal of the circuit will be denoted by a_+, the right by a_-. We shall say that

the state $A^{\langle a_m i, a_{m'} j\rangle}_{(p_1 \dots p_r)}$ is defined in the circuit if the values of the variables are determined in such a way that there is conductance between the i-th input of the m-th block and the j-th input of the m'-th block, which is destroyed when one of the contacts $p_1 p_2 \dots p_r$ is opened.

We shall say that the state $A^{\langle a_+, a_-\rangle}_{(q_1 \dots q_s)}$ includes the state $A^{\langle a_m i, a_{m'} j\rangle}_{(p_1 \dots p_r)}$ if the conductance between the poles that occurs in the state $A^{\langle a_+, a_-\rangle}_{(q_1 \dots q_s)}$ is destroyed when the contacts $p_1 \dots p_r$ are opened.

Fig. 1

Condition α**.** A boundedly block circuit satisfies condition α if there exists an r_0 (not depending on n) such that for $r \geq r_0$, for any $A^{\langle a, a_{mi}\rangle}_{(p_1 \dots p_s)}$ and any $A^{\langle a_{m+r}, a_-\rangle}_{(q_1 \dots q_t)}$ there exists a state $A^{\langle a_+, a_-\rangle}_{(p_1 \dots q_1 \dots)}$ that includes them.

To clarify the condition we give a number of examples.

Example 1 (see Fig. 1a). We number the contacts and inputs as shown in Fig. 1a on the right.

Suppose that in blocks $1, 2, \dots, m-1$ the state $A^{\langle a_+, a_{m2}\rangle}_{(k_{13} \dots k_{m-1,3})}$ is defined, and in blocks $m+r, \dots, n$ the state $A^{\langle a_{m+r}, a_-\rangle}_{(k_{m-r,1} \dots k_{n,1})}$ is defined.

Whatever r may be and however the values of the variables in blocks $m, m+1, \dots, m+r-1$ are determined, it is impossible to obtain conductance between the inputs a_{m2} and $a_{m+r,1}$, and consequently there is no state including both of the indicated states. The circuit does not satisfy condition α.

Here condition α is not satisfied because there is no chain passing through the contacts k_{i3} and k_{j1} ($i < j$). As the following example shows, condition α may also fail for another reason.

Example 2. Suppose the states shown in Fig. 1 are defined. Again there is no state $A^{\langle a+a-\rangle}_{(q_1 \dots q_s)}$ including both indicated states (any conductivity between the poles that is broken when the contacts $k_{1,5} \dots k_{l-1,5}$ and $k_{j+1,5} \dots k_{n5}$ are opened is not broken when any of the contacts $k_{l+1,5} \dots k_{m-1,5}$ and $k_{m+r,5} \dots k_{j-1,5}$ is opened). Meanwhile, here, for any pair of contacts k_{mi} and $k_{m'j}$ ($1 \leq i, j \leq 5$; $m \neq m'$), there exists a chain passing through them.

Example 3. The circuit of a parity counter (see Fig. 1). It is easy to see that the circuit satisfies condition α with $r_0 = 1$.

Theorem 1. *If a finitely block circuit satisfies condition α, then for it one can construct a unit open-circuit test of length not exceeding $c \log_2 n$ (n is the number of blocks).*

Proof. We construct a unit open-circuit test for each block. Let the test of the m-th block contain the sets of values of the circuit variables $\tilde{\alpha}_{m1} \dots \tilde{\alpha}_{mk_m}$ (obviously, $k_m \leq K$). Denote the states corresponding to these sets by $A^{\langle a_m, i^{a_m}+1, j\rangle}_{(p_1 \dots)}$ through $A_{m1} \dots A_{mk_m}$. Suppose now that there is a collection of states $A_{n_0,i_0}, A_{n_0+r_0,i_1}, \dots, A_{n_0+pr_0,i_p} \dots$.
Condition α allows one to assert that there exists a state $A^{\langle a+a-\rangle}_{(p_1 \dots q_1 \dots)}$ including all these states.

We shall assume that $k_1 = k_2 = \dots = k_m = k = \max_{1 \leq m \leq n} k_m$. This can always be achieved by denoting by $A_{m,k_m+1} \dots A_{mk}$, in those blocks where $k_m < k$, any of the states A_{mi} ($i \leq k_m$).

Let now p be an integer such that $pr_0 < n$, but $(p+1)r_0 \geq n$.

Table 1

Row no.	1	r_0+ 1	$2r_0+$ 1	…	$\left[\frac{p}{4}\right] r_0+$ 1	…	$\left[\frac{p}{2}\right] r_0+$ 1	…	$\left[\frac{3p}{4}\right] r_0+$ 1	…	pr_0+ 1
1	A_{11}	$A_{r_0+1,1}$	$A_{2r_0+1;1}$	…	$A_{[\frac{p}{4}]r_0+1;1}$	…	$A_{[\frac{p}{2}]r_0+1;1}$	…	$A_{[\frac{3p}{4}]r_0+1,1}$	…	$A_{pr_0+1,1}$
2	A_{11}	$A_{r_0+1,1}$	$A_{2r_0+1;1}$	…	$A_{[\frac{p}{4}]r_0+1;1}$	…	$A_{[\frac{p}{2}]r_0+1;1}$	…	$A_{[\frac{3p}{4}]r_0+1,2}$	…	$A_{pr_0+1,2}$
3	A_{11}	$A_{r_0+1,1}$	$A_{2r_0+1;1}$	…	$A_{[\frac{p}{4}]r_0+1;1}$	…	$A_{[\frac{p}{2}]r_0+1;2}$	…	$A_{[\frac{3p}{4}]r_0+1,2}$	…	$A_{pr_0+1,2}$
…											

Row no.	1	r_0+1	$2r_0+1$	...	$\left[\frac{p}{4}\right] r_0+1$	...	$\left[\frac{p}{2}\right] r_0+1$	...	$\left[\frac{3p}{4}\right] r_0+1$	...	pr_0+1
$s \leq [\log_2 p]+1$	A_{11}	$A_{r_0+1,2}$	$A_{2r_0+1,1}$	...	$A_{[\frac{p}{4}]r_0+1;1}$	...	$A_{[\frac{p}{2}]r_0+1;2}$	...	$A_{[\frac{3p}{4}]r_0+1,1}$	...	$A_{pr_0+1,2}$

Define the set Γ_{11} of test inputs as follows (see Table 1). The rows of the table indicate states of blocks $1, r_0+1, \ldots, pr_0+1$. In the i-th row, for the first

$$\left[\frac{p}{2^{i-1}}\right]$$

of these blocks the state A_{j1} is indicated, for the next

$$\left[\frac{p}{2^{i-1}}\right]$$

blocks—the state A_{j2}, for the next ones—again A_{j1}, and so on. $\tilde{\gamma}_i$ is such an assignment of values to the variables that corresponds to a state including all the states indicated in the i-th row of the table. The inputs $\tilde{\gamma}_1 \ldots \tilde{\gamma}_s$ form a part of the test Γ_{11}. Here the first subscript indicates that states are specified for blocks beginning with the first, and the second subscript indicates that, for the first input of the group, the states A_{j1} are specified ($j = 1, r_0+1, \ldots$). In exactly the same way we define $\Gamma_{12} \ldots \Gamma_{1k}, \Gamma_{21} \ldots \Gamma_{2k}, \ldots, \Gamma_{r_0 1} \ldots \Gamma_{r_0 k}$. It is clear that

$$\bigcup_{r=1}^{r_0} \bigcup_{j=1}^{k} \Gamma_{ij}$$

forms a test for the entire circuit. Let us estimate its length. Each set Γ_{ij} contains no more than $[\log_2 p]+1$ inputs. Hence

$$t \leq kr_0\{[\log_2 p]+1\} \leq Kr_0 \left\{\left[\log_2 \frac{r}{r_0}\right]+1\right\} \leq c \log_2 n$$

The theorem is proved.

Unit short-circuit tests

Theorem 2. *For a bounded block planar circuit, the length of a minimal unit short-circuit test is not less than n (n is the number of blocks).*

From the theorem of § 4 of [1] it follows that a short-circuit test for a planar circuit corresponds to an open-circuit test for the dual circuit.

Let a bounded block circuit S be given. Construct the dual circuit S^* for it. Denote the contacts issuing from one of the poles of S^* by $p_1, p_2, \dots, p_N$ (obviously, $N \geq n$). The open-circuit test for the circuit S^* must include at least one input on which the fault function of contact p_i is equal to zero. But whatever input is taken, on it the fault function is equal to zero for no more than one contact from $\{p_i\}$. Therefore the open-circuit test for the circuit S^* necessarily contains N inputs. Consequently, the short-circuit test of the circuit S contains at least $N \geq n$ inputs. The theorem is proved.

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Note: Figure translations are in progress. See original paper for figures.