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Abstract

Full Text

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Light Characteristics of Thick Layers of a Scattering Medium with Small Specific Absorption

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1. The exact solution of the transfer equation gives, for the brightness coefficient $R = I/I_0\omega_0$ of an arbitrary semi-infinite medium, the expression [1]

$$R_\infty(\mathbf{r}, \mathbf{r}_0) = \sum_1^\infty a_n(\mathbf{r}, \mathbf{r}_0)(1+\beta)^{-n}, \tag{1}$$

where I_0, I are the brightnesses of the illuminating and reflected light beams; $\mathbf{r}_0, \mathbf{r}$ are unit vectors of their directions; ω_0 is the solid angle of the illuminating beam; $a_n(\mathbf{r}, \mathbf{r}_0)$ depends on the form of the scattering matrix $f_{ik}(\mathbf{r}, \mathbf{r}_0)$; $\beta = \alpha/\sigma$ is the specific absorption of the medium (α, σ are the volume coefficients of absorption and scattering); n is the order of scattering. Taking into account that for $\beta \ll 1$ the mean order of scattering [2,3]

$$\bar{n} \simeq \frac{s(\mathbf{r}, \mathbf{r}_0)\eta}{2\sqrt{\beta}} \tag{2}$$

and re-expanding (1) in a series in n^{-1}, we find

$$R_\infty(\mathbf{r}, \mathbf{r}_0) \simeq \frac{\mu_0}{\pi} h(\mathbf{r}, \mathbf{r}_0) \exp[-s(\mathbf{r}, \mathbf{r}_0)\eta\sqrt{\beta}], \tag{3}$$

where $\mu = \cos\vartheta$; ϑ_0 is the angle of incidence of the illuminating beam; h, s, η depend only on the form of the scattering matrix.

Using the approximate solution of the transfer equation in the depth of a scattering medium for $\beta \ll 1$ [4], which has the form $I(\mu, \tau) = C(\mu)e^{-\gamma\tau}(1+\gamma a(\mu)+...)$, where τ is the optical depth; $\gamma \simeq \sqrt{\beta/q}$, $q = \frac{1}{2}\int_{-1}^{+1} \mu a(\mu)\, d\mu$, and $a(\mu) = a(-\mu)$ depends only on the scattering matrix, we obtain $\eta = 4\sqrt{q}$.

2. Let us consider a layer with optical thickness τ^*. If τ^* is sufficiently large, so that the diffuse illuminance E at the base of the layer is much greater than the illuminance by direct rays, then the angular distribution of the

brightness of the light emerging from the layer does not depend on τ^*, and one may put $I(\mu) = \frac{1}{\pi} g(\mu) E$, where g depends only on the scattering matrix. Introduce the notation

$$y = \eta\sqrt{\beta}; \quad x = \gamma\tau^* = \frac{y}{l}\tau^*; \quad l = 4q \tag{4}$$

and consider a layer singled out in the depth of the scattering medium. Using (3), the solution for the depth regime, and also the reciprocity and invariance theorems, we obtain expressions for the albedo R and transmittance T of the layer under quasidiffuse illumination corresponding to the conditions of the depth regime:

$$T = \frac{\operatorname{sh} y}{\operatorname{sh}(x+y)}; \quad R = \frac{\operatorname{sh} x}{\operatorname{sh}(x+y)}, \tag{5}$$

and also for the transmittance under directional illumination:

$$t \equiv \frac{I\omega}{I_0\omega_0} = \frac{\omega}{\pi}\mu_0 g(\mu_0') g(\mu) \frac{\operatorname{sh} y}{\operatorname{sh}(x+y)} + \begin{cases} \exp(-\tau^*/\mu_0)\delta_{r,\mathbf{r}_0}, & (\omega \geqslant \omega_0), \\ \dfrac{\omega}{\omega_0}\exp(-\tau^*/\mu_0)\delta_{r,\mathbf{r}_0}, & (\omega \leqslant \omega_0), \end{cases} \tag{6}$$

($\delta_{r,\mathbf{r}_0}$ is the Kronecker symbol; the last term takes into account the direct light penetrating through the layer) and for the brightness coefficient of the layer:

$$R(\mathbf{r}, \mathbf{r}_0) = \frac{\mu_0}{\pi}\left[h(\mathbf{r}, \mathbf{r}_0)\exp[-s(\mathbf{r}, \mathbf{r}_0)y] - g(\mu_0)g(\mu)\frac{e^{-x-y}\operatorname{sh} y}{\operatorname{sh}(x+y)}\right]. \tag{7}$$

3. If the layer is located on a substrate with albedo R_p and with brightness indicatrix under quasi-diffuse illumination $\dfrac{1}{\pi}C(\mu)$, then (without taking direct light into account)

$$R(\mathbf{r}, \mathbf{r}_0) = \frac{\mu_0}{\pi}\left\{h(\mathbf{r}, \mathbf{r}_0)\exp[-s(\mathbf{r}, \mathbf{r}_0)y] - g(\mu_0)g(\mu)\frac{\operatorname{sh} y}{\operatorname{sh}(x+y)}\left[e^{-x-y} - \frac{R_p \operatorname{sh} y}{\operatorname{sh}(x+y) - R_p \operatorname{sh} x}\right]\right\}, \tag{8}$$

$$t = \frac{\omega}{\pi}\mu_0 g(\mu_0)\operatorname{sh} y \left[\frac{g(\mu) - R_p\left[g(\mu) - y g(\mu)\cdot 2\int_0^1 \mu' g(\mu')G(\mu')\,d\mu'\right]}{\operatorname{sh}(x+y) - R_p \operatorname{sh} x \cdot 2\int_0^1 \mu' g(\mu')G(\mu')\,d\mu'} - \frac{\frac{1}{\pi}\int \mu' h(\mathbf{r}, \mathbf{r}')\exp[-s(\mathbf{r}, \mathbf{r}')y]G(\mu'}{\operatorname{sh}(x+y) - R_p \operatorname{sh} x \cdot 2\int_0^1 \mu' g(\mu')G(\mu} \tag{9}$$

If $G(\mu)$ does not differ greatly from

$$\overline{G}(\mu) = e^{y/2}\left[1 - \frac{y}{l}a(\mu)\right],$$

then the albedo of the layer under quasi-diffuse illumination is:

$$R = e^{-y} - \frac{\operatorname{sh} y}{\operatorname{sh}(x+y)}\left[e^{-x-y} - \frac{R_p \operatorname{sh} y}{\operatorname{sh}(x+y) - R_p \operatorname{sh} x}\right]; \tag{10}$$

the illuminance from above at the base of the layer:

$$E_\downarrow = \frac{I_0\omega_0\mu_0 g(\mu_0)\operatorname{sh} y}{\operatorname{sh}(x+y) - R_p \operatorname{sh} x}, \tag{11}$$

or, as $\beta \to 0$,

$$E_\downarrow\ I_0\omega_0\mu_0 g(\mu_0)\frac{l}{(1-R_p)\tau^* + l}; \tag{12}$$

the illuminance from above at the level x' above the base of the layer:

$$E_\downarrow(x - x') = \frac{I_0\omega_0\mu_0 g(\mu_0)\operatorname{sh}(x' - y)}{\operatorname{sh}(x+y) - \dfrac{R_p \operatorname{sh} y \operatorname{sh}(x - x')}{\operatorname{sh}(x'+y) - R_p \operatorname{sh} x'}}; \tag{13}$$

and the brightness at depth x in the half-space ($R_p = e^{-y}$):

$$I(x,\mu) = \frac{1}{\pi} I_0\omega_0\mu_0 g(\mu_0) e^{-x-y/2}\left[1 + \frac{y}{l}a(\mu)\right]. \tag{14}$$

4. The relations obtained are valid if the form of the scattering matrix does not depend on β $\left(\dfrac{\partial \ln f_{ik}}{\partial\sqrt{\beta}} \ll 1\right)$, which is certainly the case when a of the dispersing medium is varied, or when the effective radius r of the particles is varied, if $\rho \equiv 2\pi r/\lambda \gg 1$. In the latter case, for $\beta \ll 1$, $\sigma \simeq 2\pi r^2 N$ ([5]), where N is the particle concentration, and $\alpha \simeq \pi r^2 \dfrac{4\pi\varkappa}{\lambda} KrN$, where $\varkappa$ is the absorption index of the substance forming the particles, and K is the particle shape factor. Accordingly, $\beta = K\rho\varkappa$, and from (3) we obtain

$$R_\infty(\mathbf{r},\mathbf{r}_0) = \frac{\mu_0}{\pi} h(\mathbf{r},\mathbf{r}_0)\exp\left[-s(\mathbf{r},\mathbf{r}_0)\eta\sqrt{K\rho\varkappa}\right]. \tag{15}$$

If the dye is adsorbed by the particle, then $\alpha = K'\pi r^2 c \dfrac{4\pi\varkappa}{\lambda} N$, where K' is a shape factor that, for particles of irregular shape, may depend weakly on size, and c is the surface concentration of the dye. Therefore β, and consequently also R_∞, do not depend on r for spherical particles and may depend only weakly on r for particles of irregular shape.

5. Comparison of relations (3), (5), (6), (7), (12) with solutions of the transport equation on computers ([6]), and also of (3), (12), (14), and (15) with data from a number of experiments, shows good quantitative agreement for $\beta \lesssim 0.2$; $R(\tau^*, \beta) \gtrsim 0.2R(\infty, 0)$; $\tau^* \gtrsim 5\mu_0$, and 1 mm $\gtrsim r \gtrsim 1\,\mu$.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.