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Abstract

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AUTORESONANT MOTION OF A PARTICLE IN A PLANE ELECTROMAGNETIC WAVE

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The features of the motion of a charged particle in the field of a plane electromagnetic wave are well known ([1,2]). In a coordinate system in which the particle is at rest on average, it describes a trajectory in the form of a figure eight (for a plane-polarized wave) or a circle (in the case of circular polarization). In this case the energy of the particle remains constant on average, i.e., no resonant interaction with the wave occurs, and the motion, generally speaking, differs little from free motion.

In the present work we would like to draw attention to the fact that, under certain conditions—for example, in a magnetic field—the character of the motion of a particle in the field of a plane wave may change radically. In particular, it is possible to realize a resonant interaction of the wave and the particle, in which the energy of the particle will increase continuously. In principle, resonance may occur for arbitrary values of the wave frequency ω and the gyromagnetic frequency $\omega_B = eB_0/mc\gamma$. In this case the resonance will be maintained automatically, despite the increase in the energy (and mass) of the particle and the corresponding decrease in the frequency ω_B. This peculiar phenomenon we shall call autoresonance between the wave and the particle.

As the simplest and physically clearest case, let us consider the motion of a particle of rest mass m and charge e in the field of a plane wave $\mathbf{E}, \mathbf{B}$ in the presence of a homogeneous magnetic field $\mathbf{B}_0$ parallel to the wave vector $\mathbf{k}$. The components of the wave field satisfy the conditions

$$\mathbf{B} = [\mathbf{nE}], \qquad \mathbf{nE} = \mathbf{nB} = 0, \tag{1}$$

where $\mathbf{k} = k\mathbf{n}$, with $k = \omega/c$.

We shall use the dimensionless radius vector $\vec{\rho} = k\mathbf{r}$, i.e., measure distances in fractions of $\lambda/2\pi$—the wavelength divided by 2π. Taking (1) into account, and neglecting small radiation corrections, the equation of motion of the particle can be reduced to the form

$$\frac{d}{dt}\gamma\dot{\vec{\rho}} = \frac{e}{m}\left\{\left(1-\frac{\dot{\vec{\rho}}\mathbf{n}}{\omega}\right)\mathbf{E}k + \frac{\mathbf{n}}{c}(\dot{\vec{\rho}}\mathbf{E}) + \frac{B_0}{c}[\dot{\vec{\rho}}\mathbf{n}]\right\}, \tag{2}$$

where γ is the total energy in units of mc^2, and the electric field E varies proportionally to

$$\exp i(\omega t - \mathbf{kr}) = \exp i\omega\left(t - \frac{\vec{\rho}\mathbf{n}}{\omega}\right).$$

The first term on the right corresponds to the transverse force due to the electric field and the Lorentz force; the second is the longitudinal Lorentz force arising from the transverse velocity of the particle and the magnetic field of the wave; the third term is connected with the action of the external magnetic field. Let us note that from equation (2) there follows an important relation, which may be derived by multiplying (2) scalarly by $\mathbf{n}$ and integrating the resulting expression, taking into account the equality $e\dot{\vec{\rho}}\mathbf{E} = kmc^2\dot{\gamma}$. The desired relation has the form

$$\gamma\left(1-\frac{\dot{\vec{\rho}}\mathbf{n}}{\omega}\right) = \text{const} = \gamma_0(1-\beta_0), \tag{3}$$

where γ_0 corresponds to the initial energy, and β_0 to the initial value of the component of the particle velocity in the direction $\mathbf{k}$. This means that the difference between the particle energy and the longitudinal component of the momentum (in the corresponding units) is an integral of the motion.

Equation (2) can be substantially simplified if, as the independent variable, one chooses instead of time the phase of the particle relative to the wave

$$\psi = \omega t - \vec{\rho}\mathbf{n} + \psi_0, \tag{4}$$

and, most importantly, uses the relation (3) obtained above.

As a result of the change of variables, (2) takes the form

$$\vec{\rho}\,'' = \vec{\eta} + \mathbf{n}\,(\vec{\rho}\,'\vec{\eta}) + \frac{\Omega}{\omega}\,[\vec{\rho}\,'\mathbf{n}]\,, \tag{5}$$

where primes denote differentiation with respect to ψ, and the quantities Ω and $\vec{\eta}$ have the following meaning:

$$\Omega = \frac{eB_0}{mc\gamma_0(1-\beta_0)}, \qquad \vec{\eta} = \frac{\Omega\mathbf{E}}{B_0\omega} = \vec{\eta}_0\sin\psi. \tag{6}$$

Let us first consider the case of a linearly polarized wave. Let the x-axis be directed along the vector $\mathbf{n}$, the y-axis along $\mathbf{E}$, and the z-axis along $\mathbf{B}$. Equation (5), in components, takes the form:

$$\text{a) } x'' = \eta y'; \qquad \text{b) } y'' = \eta + \frac{\Omega}{\omega} z'; \qquad \text{c) } z'' = -\frac{\Omega}{\omega} y', \tag{7}$$

with

$$\gamma = \gamma_0 (1 - \beta_0)(1 + x'). \tag{8}$$

Hence follows the equation

$$y'' + \left(\frac{\Omega}{\omega}\right)^2 y = \eta_0 \sin\psi - \frac{\Omega}{\omega} z_0', \tag{9}$$

which shows that, under the condition $\Omega = \omega$, resonance occurs and the amplitude of the y-oscillations—and consequently also the quantity x'—increase linearly. Accordingly, the particle energy γ also increases. The physical meaning of this resonance, which, as is evident, is not destroyed as the particle energy grows, can be explained as follows. The frequency with which the wave field acts on the moving particle in the laboratory frame is $\omega_\beta = \omega(1 - \beta)$, where β corresponds to the longitudinal component of the particle velocity. As the energy increases, and consequently β increases, the frequency ω_β decreases. At the same time, a particle in an external magnetic field may be regarded as an oscillator performing transverse oscillations with frequency $\omega_B = eB_0/mc\gamma$, which also decreases as the energy grows. By an appropriate choice of β_0, one can make the frequencies ω_β and ω_B equal at the initial instant, i.e., obtain resonance between the wave and the particle motion, in which the energy increases at the expense of the electric field. Under the action of the magnetic field of the wave, part of the acquired momentum is transferred into longitudinal motion; moreover, as we saw above (see (3)), the product $\gamma(1 - \beta)$ remains constant. Thus, the ratio of the frequencies ω_β/ω_B is an integral of the motion, i.e., once established, the resonance is subsequently maintained despite the increase in the particle energy. The question of the stability of such motion must be considered separately.

The increase in the particle energy, which is not difficult to find using expressions (3), (4), and (9), is determined, in the most interesting case $\psi \gg 1$, by the following formula:

$$\gamma \simeq \gamma_0 + \Gamma\left[\frac{\tau^2}{8} + \frac{\beta_0 \tau}{2(1 - \beta_0)}\left(\operatorname{tg}\alpha_y \sin\psi_0 - \operatorname{tg}\alpha_z \cos\psi_0\right)\right], \tag{10}$$

where $\tau = \eta_0(\psi - \psi_0)$, and the dimensionless parameter Γ is equal to

$$\Gamma = \frac{eB_0}{mc\omega} = \frac{eB_0 \lambda}{mc^2}. \tag{11}$$

The quantities a_y and a_z are the projections of the injection angle α, which the initial momentum makes with the direction $\mathbf{k}$. It is not difficult to show that resonant motion is possible for injection angles not exceeding a certain limiting value

$$\alpha \leqslant \alpha_{\max} = \arcsin \Gamma. \tag{12}$$

For $\Gamma \geqslant 1$ this restriction becomes immaterial. At the same time, the quantity Γ also determines the admissible initial values β_0, which must satisfy the inequality

$$\Gamma^2 \geqslant \frac{1-\beta_0}{1+\beta_0}. \tag{13}$$

Thus, as Γ decreases, one must choose increasingly relativistic initial conditions in the direction $\mathbf{k}$ ($\beta_0 \to 1$), increasing the initial energy and decreasing the injection angle. Let us note, for orientation, that in the case of electrons the value of Γ for light waves is of the order $10^{-4} \div 10^{-5}$, while for centimeter waves it may exceed unity.

A particle injected into a plane wave under resonant conditions moves along a spiral with increasing radius ($R \simeq \tau/2$) and pitch, and on average increases its energy. For a given injection angle α, the resonance conditions can be realized at two different values of β_0, and correspondingly of the initial energy γ_0. In particular, for $\Gamma > 1$, one of the resulting values of β_0 is positive and the other negative. In this case one can realize a variant in which the particle flies toward the wave, resonantly gives up energy, and then is reflected and begins to accelerate in the opposite direction.

In the case of circular polarization, resonance at $\Omega = \omega$ still occurs for a wave whose electric field rotates in the same direction as the particle in the magnetic field.

The effect of autoresonant motion of a particle in a wave can in principle be applied to the acceleration of particles by means of powerful electromagnetic beams in various frequency ranges, or in the region of amplification of radio waves of various ranges. The mechanism considered may also play some role in cosmic processes, leading to the acceleration of charged particles by radio waves and light fluxes in cosmic magnetic fields.

We note that, for simplicity and to clarify the essential aspect of the question, we have considered the simplest case of wave propagation in vacuum in the presence of a homogeneous magnetic field. From the point of view of realizing an effective interaction of the wave and the particle, it may also be of interest to superpose a periodically alternating (in space) magnetic field or a time-varying electric field (with modulated frequency), etc. In addition, diverse effects of an analogous type are possible in the propagation of waves in media (isotropic, anisotropic, magnetoactive, etc.).

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.