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Abstract

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V. V. GOGOSOV

ON THE SPLITTING OF NONEVOLUTIONARY MAGNETOHYDRODYNAMIC SHOCK WAVES

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This paper proposes a method that makes it possible to determine into which combinations, consisting of evolutionary discontinuities and self-similar rarefaction waves, a nonevolutionary magnetohydrodynamic shock wave splits. No restrictions are imposed on the parameters of the medium or on the intensity of the shock wave.

Evolutionary discontinuities are discontinuous solutions of nonlinear differential equations that depend continuously on the initial and boundary conditions. Thus, gas-dynamical or magnetohydrodynamic shock waves will be evolutionary if small changes in the gas-dynamical or magnetohydrodynamic quantities cause a small change in the solution. The idea of evolutionarity was first expressed in papers ([1], [2]) in connection with the study of discontinuities in ordinary gas dynamics.

It is known that in gas dynamics the gas velocities on both sides of a shock wave satisfy the relations

$$u_1 < c_1, \qquad u_2 < c_2. \tag{1}$$

Conditions (1) are usually derived with the aid of the corresponding thermodynamic relations. However, as shown in ([1]), from considerations of evolutionarity it follows that conditions (1) are necessary for the existence of a shock wave independently of the thermodynamic properties of the gas. In magnetohydrodynamics, the requirements of evolutionarity of discontinuities impose additional conditions on magnetohydrodynamic shock waves in comparison with the conditions following from thermodynamics. These additional conditions consist in the fact that the gas velocities on both sides of magnetohydrodynamic shock waves must satisfy the relations ([3]):

$$\text{Fast shock waves:} \quad u_1 > c_{+1}, \qquad c_{+2} > u_2 > V_2. \tag{2}$$

$$\text{Slow shock waves:} \quad V_1 > u_1 > c_{-1}, \qquad c_{-2} > u_2. \tag{3}$$

Here u_1, u_2 are the components, normal to the plane of the shock wave, of the gas velocity to the left and to the right of the shock front in the coordinate system associated with the shock wave; c_+, c_- are the propagation velocities of the fast and slow magnetohydrodynamic waves; $V = H_x/\sqrt{4\pi\rho}$ is the propagation velocity of rotational discontinuities. The written conditions were derived in ([3]) for perturbations depending on x and t. Taking account of perturbations of general form, depending on x, y, z, t, leads to the same conditions of evolutionarity ([4]).

We note that in gas dynamics, if conditions (1) are not satisfied (a nonevolutionary shock wave), then on such a wave the entropy necessarily decreases (it is meant that the corresponding thermodynamic inequalities are satisfied). In magnetohydrodynamics, in nonevolutionary shock waves (inequalities (2), (3) are not satisfied) the entropy may increase. The evolutionarity of magnetohydrodynamic shock waves was also investigated in paper ([5]). (For a detailed presentation of questions connected with magnetohydrodynamic shock waves, see ([6]).) In papers ([1−5]), discontinuous solutions depending continuously on the initial and boundary conditions were called stable. The term "evolutionary" discontinuities was intro-

was considered in [7] in connection with the study of systems of quasilinear differential equations. Works [8, 9] are devoted to the same question.

For non-evolutionary discontinuities an infinitesimal perturbation causes a finite change of the solution. The discontinuity splits into several discontinuities of finite magnitude. In magnetohydrodynamics these may be shock waves, rarefaction waves, rotational and contact discontinuities. The splitting of a non-evolutionary, plane, stationary magnetohydrodynamic shock wave was considered in [10]. It was assumed that the magnetic field makes a small angle with the normal to the plane of the discontinuity, and that the magnitude of the component of the magnetic field normal to the discontinuity is such that on both sides of the discontinuity the Alfvén velocity is greater than the sound velocity,

$$V = \frac{H_x}{\sqrt{4\pi\rho}} > c = \sqrt{\frac{\gamma p}{\rho}}.$$

It is also assumed that the non-evolutionarity conditions [3, 5] are satisfied, $u_1 > V_1$, $u_2 < V_2$, and that $\sqrt{(u_1 - V_1)/V_1}$ is a small quantity. The latter requirement means that the point corresponding to such a shock wave lies in Fig. 2 of [3, 6] near the boundary of evolutionarity.

The problem is solved to first-order accuracy in small quantities. It is shown that, in this approximation, the shock wave splits into an $S^+S^-KAS^+$-combination of waves. However, from general considerations it is clear that subsequent approximations must give combinations that may include other waves. Indeed, in magnetohydrodynamics the number of independent parameters that may undergo a discontinuity on the two sides of a discontinuity is equal to 7. On each magnetohydrodynamic wave moving away from the initial discontinuity, and also on rotational and contact discontinuities, there is one free parameter; therefore, in general, 7 magnetohydrodynamic waves must move away on the two sides of the initial discontinuity. If the discontinuity is plane (as, for example, a shock wave) and if the tangential components of the magnetic field on the two sides of the discontinuity have opposite signs (a necessary condition for a non-evolutionary shock wave [6]), then the rotational discontinuity can go only in one direction, since on a contact discontinuity the magnetic fields must be equal, while the sign of the tangential component of the magnetic field can change only on a rotational discontinuity. Consequently, in general, a non-evolutionary shock wave must split into combinations consisting of 6 waves (including one rotational and one contact discontinuity).

In the present work a method is proposed that makes it possible to determine into which combinations, consisting of evolutionary discontinuities and self-similar rarefaction waves, a non-evolutionary magnetohydrodynamic shock wave splits, depending on the intensity of this non-evolutionary wave. It is shown that, indeed, in the general case a non-evolutionary shock wave splits into 6 waves.

The problem of the splitting of a non-evolutionary shock wave is a special case of the problem of the decay of an arbitrary discontinuity in magnetohydrodynamics, considered in [11], when the parameters of the medium to the left and to the right of the discontinuity are connected by the relations on a shock wave. This relation in the pH_y plane has the form [12] shown in Figs. 1 and 2. The parameters of the medium before (behind) the shock wave will be denoted by the index $0'$ (0). The bold line in Figs. 1 and 2 shows the part of the curve corresponding to the parameters on an evolutionary shock wave. The points of the thin part of the line correspond to the pressure and tangential component of the magnetic field behind a non-evolutionary shock wave. Let us emphasize, as already mentioned, that the discontinuity corresponding to a non-evolutionary shock wave is plane, i.e. $w_0 = w_0' = 0$, $H_{z0} = H_{z0}' = 0$, and the tangential component of the magnetic field vanishes or changes sign (Figs. 1 and 2).

As shown in § 11 of [11], to determine into which combinations a plane arbitrary discontinuity with $\mathbf{H}_{\tau 0} \cdot \mathbf{H}_{\tau 0}' < 0$ decays, one may use diagrams constructed for the case $\mathbf{H}_{\tau 0} \cdot \mathbf{H}_{\tau 0}' > 0$.

(Figs. 7—10, 15—18 of paper ([11])); only on these diagrams, instead of combinations without a rotational discontinuity and with two rotational discontinuities, one must substitute combinations of the same kind, but with one rotational discontinuity going to the right or to the left. It is also shown there that if

Fig. 1-3

Figure 1: Fig. 1-3

$H'_{y0} > 0$, $H_{y0} < 0$, then combinations with two rotational discontinuities (they correspond to points, lines, and regions situated above the dividing line) must be

Fig. 1 **Fig. 2** **Fig. 3**

replaced by combinations of the same kind, but with one rotational discontinuity going to the right; combinations without rotational discontinuities (these combinations correspond to points, lines, and regions lying below the dividing line) must be replaced by combinations of the same kind, but with one rotational discontinuity going to the left. We note that under such a replacement the coordinates of the lines and regions necessarily change, but the qualitative appearance of the figures remains the same. To determine which diagrams must be used in each particular case of splitting of a nonevolutionary wave, according to § 11 of paper ([11]), let us reflect the thin part of the curves in Figs. 1 and 2 symmetrically with respect to the p-axis (the dashed lines in Figs. 1 and 2). The point (p'_0, H'_{y0}) remains unchanged; the image of the point (p_0, H_{y0}) lies on the dashed line (the point (p_0, H^*_{y0})).

I. $H^*_{y0} = |H_{y0}| > H'_{y0}$ (Fig. 1). Depending on the behavior of the S^+- and R^--lines issuing from the points (p_0, H'_{y0}), (p_0, H^*_{y0}), respectively, cases described by inequalities 4.1–4.4 of paper ([11]) are possible. In order to determine into which combinations the nonevolutionary discontinuity splits, one must use the diagrams obtained by the method indicated above from the diagrams of Figs. 7–10 ([11]), respectively.

II. $H^*_{y0} = |H_{y0}| < H'_{y0}$ (Figs. 1 and 2). Apparently, in both cases the S^--line issuing from the point (p'_0, H'_{y0}) goes below the point (p_0, H^*_{y0}) in Figs. 1 and 2 (below the dashed line). Depending on the behavior of the R^--line issuing from the point (p_0, H^*_{y0}), cases described by inequalities 10.1, 10.4 of paper ([11]) are possible. In these cases, in order to determine into which combinations the nonevolutionary discontinuity splits, it is necessary to use the diagrams obtained by the method indicated above from the diagrams of Figs. 15, 18 ([11]). If the S^--line goes above or intersects the dashed line, it is necessary to use diagrams 16, 17; 15, 18 ([11]), respectively.

III. $H^*_{y0} = |H_{y0}| = H'_{y0}$ (Fig. 1). The S^+-line issuing from the point (p'_0, H'_{y0}) lies above the point (p_0, H'_{y0}); the R^+-line issuing from the point (p_0, H'_{y0}) lies below the point (p_0, H'_{y0}). The indicated case corresponds to inequality 4.1 ([11]); to determine into which combinations the discontinuity decomposes, it is necessary to use the diagram obtained by the method indicated above from the diagram of Fig. 7 ([11]). In the case under consideration of splitting of a nonevolutionary shock wave, $\Delta u = u_0 - u'_0,\ \Delta v = v_0 - v'_0$

do not pro-

arbitrary, but satisfy the relations on the nonevolutionary shock wave[1]. The relation between Δu and Δv is shown in Fig. 3 (the S^{-}-line). Therefore, not all combinations of the diagrams obtained from Figs. 7-10, 15-18 of [11] can correspond to the splitting under consideration, but only those for which the corresponding points lie on the nonevolutionary part of the S^{-}-line constructed on these diagrams.

In [11] it is shown that, if $\mathbf{H}_{\tau 0} \cdot \mathbf{H}'_{\tau 0} < 0$, then in Figs. 7-10, 15-18 the regions correspond to combinations consisting of 4 shock or rarefaction waves, one rotational and one contact discontinuity (6 waves of finite amplitude); the lines bounding these regions correspond to combinations consisting of 3 shock or rarefaction waves, one rotational and one contact discontinuity (5 waves of finite amplitude); the lines intersect at points to which there correspond combinations of 2 shock or rarefaction waves, one rotational and one contact discontinuity (4 waves of finite amplitude). It is clear from what has been said that a nonevolutionary wave splits, as a rule, into 6 waves and discontinuities, and only for certain values of the parameters into 5 or into 4. In the general case, investigation of the behavior of the S^{-}-line, representing the relation between Δu, Δv in the nonevolutionary shock wave, relative to the other lines in the diagrams obtained from Figs. 7-10, 15-18, is difficult. However, in each particular case the solution of the problem presents no difficulty.

Indeed, as soon as the numerical parameters of the medium to the left and to the right of the nonevolutionary wave are known, it is known which of inequalities 4.1-4.4; 10.1-10.4 of [11] holds, i.e., it is known which of the diagrams in Figs. 7-10, 15-18 of [11] must be used. Knowing the type of diagram and making, as indicated above, the corresponding replacements of combinations without rotational discontinuities by combinations with two rotational discontinuities by combinations with one rotational discontinuity going to the right or to the left, we know which lines must be constructed in the plane Δu, Δv. The method for obtaining the equations of these lines is indicated in [11]. Determining in which region the point Δu, Δv corresponding to the given nonevolutionary shock wave lies, we obtain into which combinations this shock wave splits.

In the investigation it was assumed that the problem of the splitting of nonevolutionary shock waves has a unique solution in the class of evolutionary waves. From this assumption it follows that evolutionary shock waves do not split. An analogous result is obtained if one formally uses the proposed apparatus to determine into which combinations an evolutionary shock wave splits. In this case it is necessary to consider the diagrams obtained from Figs. 7, 8 of [14].

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Scientific-Research Institute of Mechanics
Moscow State University

[1] Analogous relations were obtained independently by A. A. Barmin.

named after M. V. Lomonosov

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REFERENCES

1. L. D. Landau, E. M. Lifshitz, *Mechanics of Continuous Media*, Moscow, 1953.

2. R. Courant, K. Friedrichs, *Supersonic Flow and Shock Waves*, IL, 1953.

3. A. I. Akhiezer, G. Ya. Lyubarskii, R. V. Polovin, ZhETF, **35**, 731 (1958).

4. V. M. Kantorovich, ZhETF, **35**, 1216 (1958).

5. S. I. Syrovatskii, ZhETF, **35**, 1466 (1958).

6. R. V. Polovin, *Uspekhi Fizicheskikh Nauk*, **72**, issue 1 (1960).

7. I. M. Gel' fand, UMN, **14**, issue 2, 87 (1959).

8. Lax, *Comm. Pure and Appl. Math.*, **10**, 537 (1957).

9. K. I. Babenk , I. M. Gel' fand, *Scientific Reports of Higher Schools. Physical-Mathematical Sciences*, No. 1, 12 (1958).

10. G. L. Lyubarskii, R. V. Polovin, ZhETF, **36**, 1272 (1959).

11. V. V. Gogosov, *Applied Mathematics and Mechanics*, **25**, 108 (1961).

12. A. G. Kulikovskii, Dissertation, Moscow State University named after M. V. Lomonosov, 1958.

13. J. Bazer, W. B. Ericson, *Astrophys. J.*, **129**, 758 (1958).

14. V. V. Gogosov, *Applied Mathematics and Mechanics*, **25**, 187 (1961).

Note: Figure translations are in progress. See original paper for figures.