Aerodynamics

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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AN INVERTIBILITY THEOREM FOR FLOWS CLOSE TO FREE-MOLECULAR FLOWS

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Let a flat plate of arbitrary shape (Fig. 1), located in the xy plane, be flowed past by a stream of rarefied gas whose velocity vector $\vec{V}$ lies in the xz plane and makes an angle γ with the z axis.

In a free-molecular flow, the same number of molecules, imparting to each element the same momentum and energy, falls on any element of the plate of area ds. As a result of the interaction* of the molecules with the surface, the same field of reflected molecules is formed near each element of the plate, so that in any element of space near the plate there are two kinds of molecules: molecules of the incident stream and molecules reflected from the plate. The distribution function of the molecules of the incident stream near the plate differs from the distribution function at infinity, since some of the molecules are screened by the plate. Clearly, all elements of the plate screen the flow in the same way.

Fig. 1

Let us consider the change in the number of particles, momentum, and energy brought to the plate as a result of the first collisions of molecules of the incident stream with reflected molecules. It is assumed that all characteristic mean free paths (of incident molecules on reflected ones, reflected on reflected ones, etc.) are much larger than the characteristic dimension of the plate L. In this case only collisions occurring at distances of order L from the plate are significant, and, consequently, the attenuation of the incident and reflected streams as a result of collisions at these distances may be neglected; one may assume that near the plate the distribution of molecules of the incident and reflected streams is the same as in a free-molecular flow. Clearly, in this case the pattern of scattering of molecules after the first collision is a superposition of the scattering of molecules by molecules reflected from the individual elements of the plate.

As noted above, in a free-molecular flow all elements of the plate are equivalent, so that the normal and tangential momenta and the heat flux brought to the plate by the molecules are the same for a plate of arbitrary shape of given area.

In the presence of collisions, the number of particles, momentum, and energy incident on a given element of the plate differ from the corresponding quantities in a free-molecular flow. The change in the number of molecules and in the momentum and energy brought by them to the given element obviously depends on the field of ref-

* The specific law of interaction is immaterial.

of reflected molecules, which, as indicated above, is a superposition of the fields of the individual elements. Depending on the relative arrangement of the elements, the aerodynamic characteristics of the plate elements change, and consequently the aerodynamic characteristics of the entire plate of given area depend on its shape.

In accordance with what has been said, one may introduce an influence function $\vec{W}_{ij}^{k}(x_i - x_j, y_i - y_j)$ such that $ds_i \vec{W}_{ij}^{k} ds_j$ is the change in the k-th property brought to the element ds_i as a result of collisions of the molecules of the incident stream with molecules reflected from the element ds_j.

The index k denotes the property in question (normal or tangential momentum, particle flux, or energy).

The total change of the k-th property as a result of collisions is, obviously, equal to

$$\iint \iint \vec{W}_{ij}^{k} \, ds_i ds_j, \tag{1}$$

where the integration with respect to ds_i and ds_j is carried out over the entire plate.

Consider (Fig. 1), along with the direct stream, a flow which we shall call "reverse," whose velocity vector $\vec{V}$ lies in the xz-plane and makes the angle $-\gamma$ with the z axis, while $|\vec{V}| = |\vec{V}|$.

For flows of the type under consideration, a reversibility theorem holds, stating that on a plate of arbitrary shape in plan, in the direct and reverse streams, taking into account the first collisions of incident and reflected molecules, there fall the same number of molecules, the same normal momentum and energy, and tangential momentum of opposite sign.

Let us introduce the influence function in the reverse stream $\overleftarrow{W}_{ij}^{k}$. It is easy to see that

$$\overleftarrow{W}_{ij}^{k} = \overleftarrow{W}_{ji}^{k} \tag{2}$$

for the number of particles, normal momentum, and energy flux, and

$$\overleftarrow{W}_{ij}^{k} = -\vec{W}_{ji}^{k} \qquad (3)$$

for the components of the tangential momentum.

The total change of any property in the reverse stream is equal to

$$\iint\iint \overleftarrow{W}_{ij}^{k}\, ds_i ds_j \equiv \iint\iint \overleftarrow{W}_{ji}^{k}\, ds_j ds_i. \qquad (4)$$

Comparing (1) and (4), and taking into account relations (2) and (3), we are convinced of the validity of the assertion made.

In the general case, the momentum and energy brought by the molecules to the plate are also changed as a result of collisions of reflected molecules with one another. However, in most cases of interest the momentum and energy brought to the plate after such collisions are negligibly small. In particular, this occurs at hypersonic velocities of the incident stream and with an accommodation coefficient close to unity, when the velocity of the reflected molecules is much smaller than the velocity of the incident stream.

The aerodynamic characteristics of the plate depend not only on the momentum and energy brought by the molecules, but also on the momentum and energy carried away by the reflected molecules. The momentum and energy carried away by the reflected molecules are determined by the law of interaction of the molecules with the surface and by the state of the surface, in particular, by its temperature. It is obvious that, in the general case, the temperature of different surface elements is different. Therefore, from each element different momentum and energy will be carried away by reflected molecules. Their sums over the entire plate, in the general case, are also different in the direct and reverse streams.

However, there is a broad class of conditions under which the reflected molecules carry away the same momentum and energy in the direct and reverse flows. In particular, this is the case for an isothermal plate under the diffuse Maxwell law of reflection and with an arbitrary accommodation coefficient. In a number of cases, for example in the above-mentioned case of hypersonic flow, the momentum and energy of the reflected molecules may be neglected.

In those cases when the momentum and energy brought to the plate as a result of collisions of reflected molecules with one another may be neglected, and the momentum and energy carried away by the reflected molecules are determined in total over the plate by the number of incident molecules, by the momentum and by the energy, one can make a stronger assertion, namely that the aerodynamic forces and the heat flux for a plate of arbitrary shape are the same in the direct and reverse flows.

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