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# PHYSICS

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## Abstract

## Full Text

PHYSICS

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# ON A MODEL OF QUANTUM FIELD THEORY

(Presented by Academician N. N. Bogolyubov, 28 V 1962)

1. In connection with a number of difficulties in quantum field theory, the study of such model systems is of undoubted interest, since they make it possible to clarify these difficulties completely and to solve to the end the problem of the behavior of a system of interacting particles.

Following the work of <sup>(1)</sup>, let us consider one such system, described by the model Hamiltonian:

$$\begin{aligned} \mathcal{H} = & \sum_{(k)} \omega_k (b_k^+ b_k + \bar{b}_k^+ \bar{b}_k) + \sum_{(k)} (E_k - \delta M) a_k^+ a_k + \\ & + g_0 \sum_{(k,p)} \sqrt{\frac{M}{E_{k+p}}} \frac{1}{\sqrt{4\omega_k \omega_p}} (a_{k+p}^+ b_k \bar{b}_p + b_k^+ \bar{b}_p^+ a_{k+p}) + \\ & + \lambda_0 \sum_{(k,p,q)} \frac{1}{\sqrt{16\omega_k \omega_p \omega_q \omega_{k+p-q}}} b_k^+ \bar{b}_p^+ b_q \bar{b}_{k+p-q}, \end{aligned} \quad (1)$$

where

$$E_k = \sqrt{k^2 + M^2}, \quad \omega_k = \sqrt{k^2 + \mu^2}, \quad M < 2\mu, \quad (2)$$

$a_k^+, b_k^+, \bar{b}_k^+$  ( $a_k, b_k, \bar{b}_k$ ) are the creation (annihilation) operators of particles of the  $a$ -,  $b$ - and  $\bar{b}$ -types with momentum  $k$ ;  $M$  and  $\delta M$  are the observed mass and the mass renormalization of the  $a$ -particle;  $\mu$  is the observed mass of the  $b$ - and  $\bar{b}$ -particles.

The system (1) under consideration does not possess crossing symmetry either in the terms proportional to  $g_0$ , which describe in the main approximation the interaction of  $b$ - and  $\bar{b}$ -particles with the  $a$ -particle (similarly to the Lee model <sup>(2)</sup>), or in the  $\lambda_0$ -terms, which describe the scattering of  $b$ - and  $\bar{b}$ -particles by each other, since the Hamiltonian contains no terms corresponding to the transitions:

$$a + b \rightleftharpoons \bar{b}, \quad a + \bar{b} \rightleftharpoons b, \quad b + \bar{b} \rightleftharpoons \bar{b} + \bar{b}.$$

It is therefore of interest to investigate for which values of the coupling constants “ghost states” appear in the model and what their role is in models of the Lee-model type.

2. Since the operators

$$\begin{aligned} N_1 &= \sum_{(k)} a_k^\dagger a_k + \sum_{(k)} b_k^\dagger b_k, \\ N_2 &= \sum_{(k)} a_k^\dagger a_k + \sum_{(k)} \bar{b}_k^\dagger \bar{b}_k \end{aligned} \quad (3)$$

are integrals of motion, the interaction of  $a$ -,  $b$ - and  $\bar{b}$ -particles should be studied by sectors with definite fixed quantum numbers  $N_1$  and  $N_2$ . In contrast to the works <sup>(2,3)</sup>, based on the investigation of the corresponding Schrödinger equation, we shall use a more convenient method connected with the consideration of Green’s functions <sup>(4,5)</sup>. For convenience, all calculations will be carried out in the center-of-mass system.

Consider the sector  $N_1 = 1$ ,  $N_2 = 1$ . As in work <sup>(5)</sup>, we find in the  $E$ -representation the Green’s function for the  $a$ -particle:

$$\langle\langle a_0 | a_0^\dagger \rangle\rangle(E) = \frac{1}{2\pi} \frac{1}{E - M + \delta M + \frac{g_0^2 L(E)}{1 + \lambda_0 L(E)}}, \quad (4)$$

where

$$L(E) = \sum_{(\mathbf{k})} \frac{1}{(2\omega_k)^2} \frac{1}{2\omega_k - E} \quad (5)$$

and the averaging is carried out over the ground state of the Hamiltonian (1).

In view of the fact that the Green’s function for the  $a$ -particle must have a pole at the point  $E = M$ , the mass renormalization should be chosen in the form

$$\delta M = -\frac{g_0^2 L(M)}{1 + \lambda_0 L(M)}. \quad (6)$$

To describe the scattering of a  $b$ -particle by a  $\bar{b}$ -particle, we introduce the two-particle Green’s function:

$$\langle\langle b_{\mathbf{k}} \bar{b}_{-\mathbf{k}} | b_{\mathbf{p}}^+ \bar{b}_{-\mathbf{p}}^+ \rangle\rangle_{(E)}. \quad (7)$$

Taking into account that scattering can occur only in the  $S$ -state, for the scattering amplitude  $T_{(E)}$  we obtain:

$$T_{(E)} = \frac{\frac{g_0^2}{E - M + \delta M} + \lambda_0}{1 + \left( \frac{g_0^2}{E - M + \delta M} + \lambda_0 \right) L_{(E)}}, \quad (8)$$

where  $T_{(E)}$  is related to the  $S$ -wave phase in the following way:

$$T_{(E)} = -16\pi \frac{E}{\sqrt{E^2 - 4\mu^2}} e^{i\delta_{(E)}} \sin \delta_{(E)}. \quad (9)$$

Defining the renormalized charge as

$$\frac{1}{g^2} = \frac{\partial}{\partial E} \left( \frac{1}{T_{(E)}} \right) \Big|_{E=M}, \quad (10)$$

we obtain

$$\frac{1}{g^2} = \frac{A^2}{g_0^2} + I_{(M)}, \quad (11)$$

where

$$I_{(E)} = \sum_{(\mathbf{k})} \frac{1}{(2\omega_k)^2} \frac{1}{(2\omega_k - M)(2\omega_k - E)}, \quad A = 1 + \lambda_0 L_{(M)}. \quad (12)$$

Expressing  $g_0^2$  in terms of  $g^2$  and substituting into (8), we have:

$$T_{(E)} = \frac{\frac{g^2}{E - M} + \lambda_0 \frac{B}{A}}{\frac{B}{A} (1 + \lambda_0 L_{(E)}) + g^2 I_{(E)}}, \quad (13)$$

where

$$B = 1 - g^2 I_{(M)}. \quad (14)$$

The structure of formula (13) makes it possible, in a natural way, to introduce one more renormalized charge  $\lambda$ , which should be defined as

$$\lambda = \frac{B}{A} \lambda_0. \quad (15)$$

Then  $T_{(E)}$  will take the form:

$$T_{(E)} = \frac{\frac{g^2}{E-M} + \lambda}{1 + (E-M) \sum_{(k)} \frac{1}{(2\omega_k)^2} \frac{1}{(2\omega_k - M)(2\omega_k - E)} \left[ \frac{g^2}{2\omega_k - M} + \lambda \right]}. \quad (16)$$

The scattering amplitude obtained for  $\lambda > 0$  and any finite  $g^2$  (with the exception of the special case  $g^2 I_{(M)} = 1$ ) has an additional pole on the real semiaxis of energies  $E < M$ . Moreover, from relations (11) and (15) it is clear that, for nonzero bare charges  $g_0$  and  $\lambda_0$ , the renormalized charges  $g$  and  $\lambda$  vanish as a consequence of the logarithmic divergence of  $L_{(E)}$ . Thus, in the given model, the double breaking of crossing symmetry corresponds to the vanishing of both the charge  $g$  and the charge  $\lambda$ .

It is interesting to note that, unlike the relativistic model <sup>(1)</sup>, the scattering amplitude as  $E \rightarrow -\infty$  tends to the finite limit  $\lambda_0$ . As was to be expected <sup>(6)</sup>, the scattering amplitude has a zero at the point

$$E_0 = M - \frac{g^2}{\lambda}, \quad (17)$$

coinciding with the bare mass of the  $a$ -particle.

The scattering of a  $b$ -particle by a  $\bar{b}$ -particle can also be studied by the determinant method <sup>(1)</sup>. Since the system under consideration is solved exactly, the principal approximation of the determinant method turns out to be the exact solution, provided only that the bare constants are replaced by the renormalized ones.

3. The Hamiltonian (1) for  $\lambda_0 = 0$  corresponds to Lee' s boson model with recoil. In this case, for the scattering amplitude we obtain:

$$T_{(E)} = \frac{g_0^2}{E - M + \delta M + g_0^2 L_{(E)}}. \quad (18)$$

In contrast to Lee' s model with a fixed source, here there is only a logarithmic divergence, which is compensated by choosing the mass renormalization in the form

$$\delta M = -g_0^2 L_{(M)}, \quad (19)$$

and a finite charge renormalization

$$g^2 = \frac{g_0^2}{1 + g_0^2 I_{(M)}}. \quad (20)$$

Owing to recoil, the critical value of the coupling constant is

$$g_{\text{cr}}^2 = \frac{1}{I_{(M)}} > 0. \quad (21)$$

Therefore the scattering amplitude, written in terms of the renormalized charge,

$$T_{(E)} = \frac{g^2}{(E - M) \left\{ 1 + g^2 (E - M) \sum_{(k)} \frac{1}{(2\omega_k)^2} \frac{1}{(2\omega_k - M)^2 (2\omega_k - E)} \right\}} \quad (22)$$

determines the phase of the  $S$ -wave for any  $g^2$ , since for any bare charge  $g_0$  the renormalized charge is

$$g^2 = \frac{g_0^2 g_{\text{cr}}^2}{g_0^2 + g_{\text{cr}}^2} < g_{\text{cr}}^2, \quad (23)$$

and, consequently, there are no additional poles on the real half-axis of energies  $E < M$ .

The case considered differs substantially from the Lee model with a fixed source, where, in the absence of a cut-off,  $g_{\text{kp}}^2$  tended to zero and therefore a “ghost state” appeared, corresponding to an extraneous pole of the scattering amplitude (or of the distribution function of the  $a$ -particle) at some real value  $E < 0$ .

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*Note: Figure translations are in progress. See original paper for figures.*

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