Reports of the Academy of Sciences of the USSR

Academician I. I. ARTOBOLEVSKII

1962

SovietRxiv

Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

Reports of the Academy of Sciences of the USSR
1962. Vol. 145, No. 3

MECHANICS

Academician I. I. ARTOBOLEVSKII

THE CURVATURE OF LINEARLY ENVELOPING CYCLOIDAL CURVES

In our work ([1]) a general theory of linearly enveloping cycloidal curves was set forth, and their parametric equations were obtained. In the present work the curvature of these curves is investigated.

Consider a mechanism (Fig. 1) consisting of a fixed circular wheel 2, a moving circular wheel 1, and a link 3, forming fifth-class pairs with wheels 1 and 2. Let an arbitrarily chosen straight line t—t reproduce a linearly enveloping cycloidal curve ([1]). Rotate link 3 through an angle Φ into position $3'$. Then wheel 1 will rotate through an angle ψ and pass into position $1'$, while the straight line t—t will pass into position t'—t'. Since wheel 1 is the moving centrode C , and wheel 2 the fixed centrode C , the point P'_{12} will be the instantaneous center of rotation in the motion of the centrode C over C in the position under consideration. Drop from the point P'_{12} the perpendicular $P'_{12}M$ to the straight line t'—t'. The point M will belong to the linearly enveloping cycloidal curve. To determine the radius of curvature ρ_M of the linearly enveloping curve at the point M, one may use the Bobillier construction ([2]). From the point O'_C draw a straight line perpendicular to the line t'—t' up to its intersection at the point D with the perpendicular erected at the point P'_{12} to the normal $P'_{12}n$. Join the obtained point D with the point O_C. The point O of intersection of the straight line DO_C with the normal $P'_{12}n$ will be the center of curvature of the linearly enveloping cycloidal curve at the point M. The segment OM will be the radius of curvature at this point.

Fig. 1

From Fig. 1 we have

$$\rho_M = P'_{12}O + R_1 \cos\varphi + a, \tag{1}$$

where a is a constant segment, equal to the shortest distance from the line t'—t to the center O'_C, namely $a = O'_C N'$.

From the Euler-Savary equation ([2]) we have

$$P'_{12}O = \frac{R_1 R_2}{R_1 + R_2}\cos\varphi = R_2\frac{\lambda}{1+\lambda}\cos\varphi, \tag{2}$$

where $\lambda = R_1/R_2$. Then equation (1) will have the form

$$\rho_M = kR_2\cos\varphi + a, \tag{3}$$

where $k = \lambda(2+\lambda)/(1+\lambda)$.

It follows from Fig. 1 that the constant quantity kR_2 is equal to the segment $O'_{C_p}A$. Then the constant segment $O_{C_n}A$ will be equal to

$$O_{C_n}A = (R_1 + R_2) - O'_{C_p}A = mR_2, \tag{4}$$

where $m = 1/(1+\lambda)$.

It follows from equation (4) that, in order to determine the center of curvature O of the linearly enveloping curve at the point M, one must lay off from the point O_{C_n} the constant segment $O_{C_n}A$, determined by equation (4), and orthogonally project the resulting point A onto the normal $P'_{12}n$. The locus of the points A will be a circle. Thus, determination of the radius of curvature ρ_M and of the center of curvature can be carried out without using Bobillier' s construction, and much more simply by orthogonal projection of the point A, lying on link 3 at the constant distance $O_{C_n}A$ determined by equation (4). The analytic expression for the radius of curvature ρ_M as a function of the angle θ (Fig. 1), entering into the parametric equations $x = x(\theta)$ and $y = y(\theta)$ of the linearly enveloping curve ([1]), can be obtained from the condition (Fig. 1) that the angle φ is equal to

$$\varphi = 180^\circ - \frac{\beta-\theta}{1+\lambda}, \tag{5}$$

where β is a constant angle (Fig. 1) determining the initial position of the mechanism.

Then the equation for ρ_M in parametric form $\rho_M = \rho_M(\theta)$ will have the form

$$\rho_M = -kR_2\cos m(\beta-\theta) + a. \tag{6}$$

In some cases it is convenient to have the equation for ρ_M as a function of the angle Φ of rotation of link 3, i.e., the equation in the form

Fig. 2

Figure 2: Fig. 2

$$\rho_M = kR_2 \cos\frac{1}{\lambda}\Phi + a, \tag{7}$$

since the angle Φ is equal to $\Phi = \dfrac{\beta - \theta}{1 + \lambda}\lambda$. The coordinates x_0, y_0 of the center of curvature O, as functions of the angle Φ of rotation of link 3, are determined from the equations

$$x_0 = mR_2 \left[\cos(\alpha - \Phi) \mp \frac{1 - m}{m} \sin\frac{m}{1 - m}\Phi \sin\left(\alpha - \frac{1}{1 - m}\Phi\right)\right], \tag{8}$$

$$y_0 = mR_2 \left[\sin(\alpha - \Phi) \pm \frac{1 - m}{m} \sin\frac{m}{1 - m}\Phi \cos\left(\alpha - \frac{1}{1 - m}\Phi\right)\right], \tag{9}$$

where $m = 1/(1 + \lambda)$. In equations (8) and (9), for external gearing the upper signs of the second term in the square brackets should be taken, and for internal gearing the lower signs.

Determination of the radius of curvature ρ_M, the center of curvature O, and the coordinates x_0 and y_0 of the center of curvature for a mechanism with internal gearing (Fig. 2)

can be done by formulas (3), (4), (6), (7), (8), and (9), taking in them the coefficients k and m equal to $k = \lambda(2 - \lambda)/(1 - \lambda)$ and $m = 1/(1 - \lambda)$.

Figure 2 shows the geometric construction for finding the center of curvature O and the radius of curvature ρ_M for the case when the segment a is equal to zero. For this, the point A, lying on the circle of radius R_A, equal to

$$R_A = mR_2 = \frac{1}{1 - \lambda}R_2,$$

must be orthogonally projected onto the normal $P_{12}n$ to the linearly enveloping curve reproduced by the line t—t. The segment OM will be the radius ρ_M of curvature of the linearly enveloping curve, and the point O will be the center of curvature of this curve at the point M.

Fig. 2

We proceed to consider the curvature of the linearly enveloping curve reproduced by the line t—t (Fig. 3), belonging to wheel 1, rolling without slipping along the fixed straight line 2.

Fig. 3

Figure 3: Fig. 3

As was shown earlier ([1]), it is sufficient to consider the case when the line t—t (Fig. 3), belonging to the moving centrode C_p, passes through the center O_{C_p} of this centrode. The position of the center of curvature O will be found,

Fig. 3

if the point A, lying on link 3 at the distance $O_{C_p}A = 2R_1$, is projected orthogonally onto the corresponding direction of the normal $P_{12}n$ of the linearly enveloping cycloidal curve reproduced by the line t—t.

For the proof we shall use the Bobillier construction ([2]). Through the point O_{C_p} draw the straight line $O_{C_p}D$, parallel to the normal $P_{12}n$, until its intersection at point D with the perpendicular erected at point P_{12} to the normal $P_{12}n$. The position of the center of curvature O of the line-enveloping curve is determined if, through point D, the line DO is drawn parallel to the line $O_{c_\text{p}}P_{12}$, up to its intersection at point O with the normal $P_{12}n$. From Fig. 3 it follows that

$$\rho_M = P_{12}O + R_1 \cos\varphi. \tag{10}$$

From the Euler-Savary equation ([2]) we obtain

$$P_{12}O = R_1 \cos\varphi. \tag{11}$$

Consequently, the radius of curvature ρ_M is equal to

$$\rho_M = 2R_1 \cos\varphi, \tag{12}$$

i.e., the construction proposed by us is satisfied.

Let us lay off on link 3 the segment $O_{c_\text{p}}A$, equal to $2R_1$. The position of the point O in each position of the mechanism is determined if the point A, describing the straight line $a - a$, parallel to line 2, is orthogonally projected onto the corresponding position of the normal $P_{12}n$.

The coordinates x_0 and y_0 of the center of curvature O as a function of the angle of rotation φ of wheel 1 can be determined from the following considerations. Let us choose the origin of coordinates at the point P_{12}^0, which is the instantaneous center of rotation of wheel 1 in the position when the line $t - t$ is parallel to line 2 and the center O_{c_p} of wheel 2 is in the position $O_{c_\text{p}}^0$. From Fig. 3 it follows that

$$x_0 = s - AO\cos\varphi = -R_1(\varphi - \sin\varphi\cos\varphi), \quad (13)$$

$$y_0 = R_1 - AO\sin\varphi = -R_1\cos^2\varphi, \quad (14)$$

since $s = R_1\varphi$.

The equation of the evolute of the line-enveloping curve formed by the line $t-t$ (Fig. 3), in explicit form, is

$$x_0 = R_1 \arccos\sqrt{\frac{y_0}{R_1}} \pm \sqrt{(R_1 - y_0)y_0}. \quad (15)$$

The proposed method can also be used for line-enveloping cycloidal curves obtained by three-link and planetary mechanisms with noncircular wheels.

Received
6 IV 1962

CITED LITERATURE

1. I. I. Artobolevskii, *DAN*, **144**, No. 1 (1962).
2. I. I. Artobolevskii, N. I. Levitskii, S. A. Cherkudinov, *Synthesis of Planar Mechanisms*, Moscow, 1959.

Note: Figure translations are in progress. See original paper for figures.