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Abstract

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THE INFLUENCE OF MAGNETIC FIELDS ON THE SHAPE OF PLANETARY NEBULAE

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Planetary nebulae are expanding gaseous spheres, i.e., they possess positive total energy. If a nebula has its own magnetic field, then it will play the same role in questions of the mechanical equilibrium of the nebula as the kinetic energy; the magnetic energy, being positive in sign, will oppose the potential energy, which has a negative sign, and will tend to disturb the equilibrium, i.e., will lead to the expansion of the nebula. Let us first determine at what values of the field strength this can occur, if the dimensions and mass of the nebula are known.

The problem posed is most easily solved with the aid of the virial theorem. In its most general form the corresponding expression for this theorem was derived by Chandrasekhar and Fermi (^{1}) and has the form:

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2T + 3(\gamma - 1)U + \mathfrak{M} + \Omega, \tag{1}$$

where I is the moment of inertia of the mass of the gaseous sphere; T, U, $\mathfrak{M}$, and Ω are, respectively, its kinetic, thermal, magnetic, and potential energies; γ is the adiabatic exponent.

The moment of inertia I changes with the expansion of the nebula. Taking the expansion velocity to be constant and equal to v_0, for a homogeneous spherical nebula of mass M_0 we shall have:

$$I(t) = \int_0^{M_0} r^2\,dm = \frac{3}{5}M_0 v_0^2 t, \tag{2}$$

where t is the time during which the radius of the nebula reaches the value $R(= v_0 t)$. Substituting the value of I from (2) into (1) and taking into account that $\frac{1}{2}M_0 v_0^2 = T$, we obtain:

$$^4/_5\,T + 3(\gamma - 1)U + \mathfrak{M} + \Omega = 0. \tag{3}$$

On the other hand, for the total energy of the nebula E (neglecting radiative energy), we have

$$E = T + U + \mathfrak{M} + \Omega. \tag{4}$$

Eliminating U from this by means of (3), we obtain:

$$E = -\frac{3\gamma - 4}{3(\gamma - 1)}\left(|\Omega| - \mathfrak{M}\right) + \frac{15\gamma - 19}{15(\gamma - 1)} T. \tag{5}$$

In order for the nebula not to expand, it is necessary that its total energy be negative, i.e., $E < 0$. Then from (5), also substituting $\gamma = 5/3$ (hydrogen), we shall have

$$^{6}/_{5}\, T + \mathfrak{M} < |\Omega|. \tag{6}$$

In the case when $T = 0$, the condition for preserving equilibrium of the nebula takes the form $\mathfrak{M} < |\Omega|$. But planetary nebulae expand, i.e., they have no mechanical equilibrium. Assuming for the time being that this is caused only by the action of the nebula’ s magnetic field, we shall have

$$\mathfrak{M} > |\Omega|. \tag{7}$$

For spherical configurations with constant matter density we have

$$|\Omega| = \frac{3}{5}\frac{GM_0^2}{R}; \tag{8}$$

$$\mathfrak{M} = \frac{1}{8\pi}(\overline{H})^2 V = \frac{1}{6}(\overline{H})^2 R^3, \tag{9}$$

where V is the volume of the nebula, and $\overline{H}$ is the mean value of the field strength in it. Substituting (8) and (9) into (7), we find

$$\overline{H} > \left(\frac{18}{5} G\right)^{1/2} \frac{M_0}{R}. \tag{10}$$

For a typical planetary nebula we have $M_0 \sim 0.1\odot$, $R \sim 10000$ AU, which gives

$$\overline{H} > 10^{-6} \text{ gauss}. \tag{11}$$

This result is of interest above all because it indicates the important role of the magnetic field in the question of the mechanical equilibrium of the nebula. Even in the absence of the other types of forces that lead to the expansion of the nebula (inertial forces, gas and radiation pressure), a magnetic field with $\overline{H} > 10^{-6}$

gauss alone is sufficient for the nebula, having lost mechanical equilibrium, to be in a state of expansion. Earlier, proceeding from the idea that in planetary nebulae there are magnetic fields of a nonpoint dipole type, it was found for the middle parts of nebulae that $\overline{H} \sim 10^{-3}$–$10^{-4}$ gauss. The observed bipolarity of many nebulae is caused by dipole fields ([2,3]).

The considerations set forth, although they make it possible to determine the lower value of the field strength in nebulae, at the same time serve as the starting point for solving our main problem—the clarification of the influence of planetary nebulae' s own magnetic fields on their shape.

If the magnetic field of a nebula increases its kinetic energy, then it should be thought that, in the presence of such a field, the nebula must expand with a somewhat greater velocity than when this field is absent. In the case of a homogeneous magnetic field, the additional expansion velocity imparted by it would have the same magnitude in all directions, and the external form of the nebula would remain unchanged; a nebula initially spherical would remain spherical all the time. The situation is quite different when an inhomogeneous, for example dipole, field is present in the nebula.

Let us denote by T_0 the intrinsic kinetic energy of expansion of a unit volume of the nebula; T_0 is constant in all directions and at a given stage of expansion. If a dipole magnetic field is also superposed on the nebula, then the total energy of a unit volume will increase at the expense of the magnetic energy $\mathfrak{M}(r, \varphi)$ and will be $T_0 + \mathfrak{M}(r, \varphi)$. To determine the expansion velocity of the nebula $v(r, \varphi)$ in a given direction, we have

$$\frac{\rho v^2(r, \varphi)}{2} = T_0 + \mathfrak{M}(r, \varphi), \tag{12}$$

where ρ is the gas density.

The dipole magnetic field reaches its maximum value at the poles and its minimum at the equator. Therefore the expansion velocity of the nebula···

···in the presence of a dipole field in it will be greatest in the directions of the poles and smallest in the direction of the equator. The difference in expansion velocities will lead to the initially spherical nebula taking on an elongated form—it should stretch out along the magnetic axis. Obviously, the degree of elongation will be the greater, the greater the magnetic energy in comparison with its own kinetic energy.

Here only a qualitative picture of the phenomenon has been described. In an exact quantitative treatment of the problem it would be necessary to write down and solve the corresponding equations of magnetohydrodynamics for a moving medium. However, such an attempt should be regarded as somewhat premature, if only because we do not know how the totality of magnetic lines of force changes and, in particular, how the size of the dipole l changes with the expansion of the nebula (see $(^2)$). Therefore we shall restrict ourselves to considering a certain approximate solution of the problem.

For the magnetic energy per unit volume at the outer boundaries of the nebula we have $\mathfrak{M}(r,\varphi) = \frac{a^2}{8\pi}\eta_1^2(r,\varphi)$, where a is the magnetic moment of the dipole; $\eta_1(r,\varphi)$ is a certain function of the polar coordinates r and φ of a point in the nebula; graphs of this function are given in $(^2)$. Substituting this into (12), we find:

$$\left[\frac{v(r,\varphi)}{v_0}\right]^2 = 1 + \frac{a^2}{8\pi T_0}\eta_1^2(r,\varphi), \tag{13}$$

where v_0 is the expansion velocity of the nebula in the absence of a magnetic field. At the center of the nebula $\mathfrak{M}_0 = \frac{a^2}{8\pi}\eta_1^2(0,0)$. Then we shall have, instead of (13),

$$\frac{v(r,\varphi)}{v_0} = \left[1 + q\eta_1^2(r,\varphi)\right]^{1/2}, \tag{14}$$

where

$$q = \frac{\mathfrak{M}_0}{T_0}\frac{1}{\eta_1^2(0,0)}.$$

Fig. 1

The extent of the nebula (more precisely, of a certain thin layer of it located at a distance r from the center) in a given direction, i.e. $R(\varphi)$, will obviously be proportional to $v(r,\varphi)$. Therefore, for determining the outer outline of the nebula, we have:

$$R(\varphi) \sim \left[1 + q\eta_1^2(r,\varphi)\right]^{1/2}. \tag{15}$$

Thus, the shape of the nebula in the presence of a dipole magnetic field depends on q, i.e. on the ratio of the magnetic energy at the center of the nebula to its own kinetic energy, and also on the degree of "embedding" of the magnetic poles, i.e. on the form of the function $\eta_1(r,\varphi)$.

In Fig. 1 are shown schematic contours of planetary nebulae constructed using relation (15) for various values of q (from $q = 0$ to $q = \infty$) and three values of the ratio r/l, 0.5; 1 and 2, characterizing different degrees of "embedding" of the magnetic poles in the nebula. The case $q = \infty$ means that the shape of the nebula is determined entirely magnetic field. For $q = 0$ the magnetic field is absent, and therefore the nebula has a spherical shape. Further, the larger r/l is, the deeper the magnetic poles are located (counting from the outer boundary of the nebula). At $r/l = 2$ the dipole is practically a point one.

It follows from the figure that, in fact, when dipole magnetic fields of different parameters are present in nebulae, their external form can vary within very wide limits. There are nebulae of the "hourglass" type ($a1, a2, b1$), "gymnastic kettlebells" ($c1$), "rectangular" ($b2$), ellipsoidal ($b3, c2$) and, finally, almost spherical ($c3$). In all cases the nebulae must inevitably be simultaneously bipolar. Hence we arrive at the conclusion that bipolar nebulae are characterized by the most diverse external forms, ranging from almost rectangular to almost spherical. An example of nebulae of the first type is IC 4406 (^{4}), and of the second type NGC 2474–75 (^{5}).

Thus, the hypothesis that dipole magnetic fields are present in planetary nebulae can explain not only the bipolarity of their structure, but also the elongation and the observed diversity of their external forms.

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