STRESSES AND STRAINS IN A CYCLICALLY HARDENING MEDIUM

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Abstract

Full Text

THEORY OF ELASTICITY

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STRESSES AND STRAINS IN A CYCLICALLY HARDENING MEDIUM

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The problem considered is that of determining the stress tensor with components $\sigma_{ij}^{(n)}$ and the strain tensor $\varepsilon_{ij}^{(n)}$ under any n-th loading by forces P_i (body forces) and $P_{\nu i}$ (surface forces), it being assumed that the external forces vary according to the law of symmetric cycles or cycles close to symmetric ones, and that P_i and $P_{\nu i}$ denote their greatest values in modulus. In addition, it is assumed that the elastic-plastic medium possesses the property of cyclic hardening, consisting in the fact that in cycles with a prescribed stress amplitude a decrease of the plastic components of strain is observed as the number of loadings increases.

If the plastic components of the strains $\varepsilon_{ij}^{(n)}$ are denoted by $\varepsilon_{ijp}^{(n)}$, then

$$\varepsilon_{ij}^{(n)} = \frac{1}{2G}\left(\sigma_{ij}^{(n)} - \frac{\sigma_{kk}^{(n)}\delta_{ij}}{1+m}\right) + \varepsilon_{ijp}^{(n)} \quad (i,j = x,y,z), \tag{1}$$

where G is the shear modulus and m is Poisson' s number.

The equilibrium equations and boundary conditions must be satisfied:

$$\frac{\partial\sigma_{ij}^{(n)}}{\partial j} + P_i = 0, \qquad \sigma_{ij}^{(n)} l_j = P_{\nu i}. \tag{2}$$

For the first loading,

$$\varepsilon'_{ij} = \frac{1}{2G}\left(\sigma'_{ij} - \frac{\sigma'_{kk}\delta_{ij}}{1+m}\right) + \varepsilon'_{ijp}, \tag{3}$$

$$\frac{\partial\sigma'_{ij}}{\partial j} + P_i = 0, \qquad \sigma'_{ij} l_j = P_{\nu j}. \tag{4}$$

The dependence of the intensity of the plastic strains $\varepsilon_p^{(n)}$ on the number of loadings n for a given material can be determined experimentally by recording stress-strain diagrams under symmetric loading cycles in tests with the simplest homogeneous stressed states. Since, however, after a significant number of loadings in the case of a cyclically hardening material a limiting state ([1]) is reached, for which $\varepsilon_{up}^{(\infty)} = \beta\varepsilon_{up}$ ($\beta = \text{const},\ 0 \le \beta \le 1$), one may assume that for any n

$$\varepsilon_{up}^{(n)} = \varepsilon_{up}\,[\beta + (1-\beta)f(n)]. \tag{5}$$

In this case it is necessary to require that $f(1) = 1,\ f(\infty) = 0$.

The fact that $\varepsilon_{up}^{(n)}$ is proportional to ε_{up} for different n is confirmed by the experimental results presented in Fig. 1. They are satisfactorily described by formula (5) for $f = 1/\sqrt{n}$. (L. Coffin ([2]) arrived at an analogous relation when observing the connection between a certain conventional magnitude of plastic strain and the number of cycles to failure.)

If the external forces vary throughout the entire loading process proportionally to one common parameter, then for any n-th loading there is simple loading ([3]), a consequence of which is the proportionality of $\varepsilon_{ijp}^{(n)}$ and ε'_{ijp}, with all components having one and the same proportionality factor ···

tionality. In this case it follows from (5) that

$$\varepsilon_{ijp}^{(n)} = \psi(\beta,n)\,\varepsilon'_{ijp} \quad (i,j = x,y,z), \tag{6}$$

where

$$\psi(\beta,n) = \beta + (1-\beta)f(n). \tag{7}$$

If we introduce the notation

$$\begin{aligned} \sigma_{ij}^{(n)} - \psi(\beta,n)\sigma'_{ij} &= [1-\psi(\beta,n)]\sigma_{ij}^{(e)}, \\ \varepsilon_{ij}^{(n)} - \psi(\beta,n)\varepsilon'_{ij} &= [1-\psi(\beta,n)]\varepsilon_{ij}^{(e)}, \end{aligned} \tag{8}$$

then from (1) and (3), under condition (6), it follows that the quantities $\sigma_{ij}^{(e)}$ and $\varepsilon_{ij}^{(e)}$ are related by the generalized Hooke law:

$$\varepsilon_{ij}^{(e)} = \frac{1}{2G}\left(\sigma_{ij}^{(e)} - \frac{\sigma_{kk}^{(e)}\delta_{ij}}{1+m}\right). \tag{9}$$

The equilibrium equations and boundary conditions [(2) and (4)], according to (8), reduce to the form

$$\frac{\partial\sigma_{ij}^{(e)}}{\partial j} + P_i = 0, \qquad \sigma_{ij}^{(e)} l_j = P_{\nu i}. \tag{10}$$

The components $\varepsilon_{ij}^{(n)}$ and ε'_{ij} satisfy the compatibility conditions for strains; therefore the quantities $\varepsilon_{ij}^{(e)}$ will also satisfy these conditions. This conclusion, together with relations (9) and (10), makes it possible to conclude that $\sigma_{ij}^{(e)}$ and $\varepsilon_{ij}^{(e)}$ are the stresses and strains arising in the body under consideration, which is subjected to the action of the forces P_i and $P_{\nu i}$, under the condition that the material is ideally elastic.

Fig. 1. Dependence of plastic strains under the n-th and first loadings (AK-6 alloy): a –for $n = 5$, b –for $n = 100$ (ε_p in %)

From (8), taking (7) into account, we obtain

$$\begin{aligned} \sigma_{ij}^{(n)} &= (\beta + (1-\beta) f(n))\sigma'_{ij} + [(1-\beta)(1-f(n))]\sigma_{ij}^{(e)}, \\ \varepsilon_{ij}^{(n)} &= (\beta + (1-\beta) f(n))\varepsilon'_{ij} + [(1-\beta)(1-f(n))]\varepsilon_{ij}^{(e)}. \end{aligned} \tag{11}$$

Thus, if the stresses σ'_{ij} and strains ε'_{ij}, which arise during the first elastic-plastic loading by the forces $P_i,\ P_{\nu i}$, and the corresponding stresses $\sigma_{ij}^{(e)}$ and strains $\varepsilon_{ij}^{(e)}$ for an ideally elastic material are known, then formulas (11) make it possible to determine the stresses $\sigma_{ij}^{(n)}$ and strains $\varepsilon_{ij}^{(n)}$ arising under any n-th loading by the forces $P_i,\ P_{\nu i}$. As $n \to \infty$, theorem on the limiting state follows from (11) ([1]).

Formulas (11), in the case of complete plasticity, may also be used for a cyclically softening medium. In this case $\beta > 1$.

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Note: Figure translations are in progress. See original paper for figures.