V. P. SILIN and L. M. GORBUNOV

1962

SovietRxiv

Abstract

Full Text

V. P. SILIN and L. M. GORBUNOV

ON THE KINETICS OF A NONISOTHERMAL PLASMA

(Presented by Academician I. E. Tamm on 17 February 1962)

Progress achieved recently in developing the foundations of the kinetic theory of systems of charged particles has led to the derivation of a collision integral for plasma which, owing to a consistent allowance for the polarization of the medium, makes it possible, simultaneously with collisions, to consider interaction with plasma waves. In the present communication an important influence of the interaction with such waves on the transport coefficients of a nonisothermal plasma is revealed.

High-frequency Langmuir waves, under conditions in which they exist, have a phase velocity considerably exceeding the velocity of the plasma particles. Therefore the contribution of such waves to the kinetic coefficients is small. Sound waves in an isothermal plasma practically do not exist. In a nonisothermal plasma, under the condition that the electron temperature T_e considerably exceeds the ion temperature T_i, weakly damped sound waves are possible. The velocity of such waves turns out, as is known ([1]), to be much greater than the thermal velocity of the ions. Therefore the ions interact only weakly with the sound waves. This is also clear from the fact that the damping of such sound waves is determined by the electrons, whose thermal velocity is greater than the speed of sound. From all that has been said it follows that, in order to ascertain the role of interaction with plasma oscillations and to determine the corresponding influence on the kinetic coefficients of a nonisothermal plasma ($T_e \gg T_i$), it is sufficient to take account of plasma-polarization effects only in collisions of electrons with one another; for the description of collisions of electrons with ions and of ions with ions one may use the ordinary collision integral, in which polarization is neglected. It is also clear from the above that only the contribution to the kinetic coefficients due to the electron distribution function is modified. Therefore it is sufficient to confine ourselves to consideration of a single kinetic equation for the electrons. We shall not at first consider the viscosity tensor; therefore the left-hand side of the kinetic equation may be written in the form

$$\mathbf{v}_\alpha \left(\frac{\partial \ln(N_\alpha T_\alpha)}{\partial \mathbf{r}_\alpha} - \frac{e_\alpha \mathbf{E}}{\varkappa T_\alpha} + \left[\frac{p_\alpha^2}{2m_\alpha \varkappa T_\alpha} - \frac{5}{2} \right] \frac{\partial \ln T_\alpha}{\partial \mathbf{r}_\alpha} \right) f_{\alpha 0}, \tag{1}$$

where $f_{\alpha 0}$ is the Maxwellian distribution.

We write the collision integral taking account of plasma polarization in the form ([2])

$$-\frac{\partial}{\partial p_\alpha^i}\sum_\beta \int dp_\beta\, I_{\alpha\beta}^{ij}(\mathbf{v}_\alpha,\mathbf{v}_\beta)\left[\frac{\partial f_\alpha}{\partial p_\alpha^j} f_\beta - f_\alpha \frac{\partial f_\beta}{\partial p_\beta^j}\right], \tag{2}$$

where

$$I_{\alpha\beta}^{ij}(\mathbf{v}_\alpha,\mathbf{v}_\beta) = \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{k^i k^j \pi\delta(\mathbf{k}\mathbf{v}_\alpha - \mathbf{k}\mathbf{v}_\beta)}{\varepsilon^+(\mathbf{k}\mathbf{v}_\alpha,\mathbf{k})\,\varepsilon^-(\mathbf{k}\mathbf{v}_\alpha,\mathbf{k})}\left(\frac{4\pi e_\alpha e_\beta}{k^2}\right)^2. \tag{3}$$

$$\varepsilon^\pm(\omega,\mathbf{k}) = 1 + \sum_\alpha \frac{4\pi e_\alpha^2}{k^2}\int \frac{dp_\alpha}{\omega \pm i\Delta - \mathbf{k}\mathbf{v}_\alpha}\left(\mathbf{k}\frac{\partial f_\alpha}{\partial p_\alpha}\right), \tag{4}$$

Δ is an infinitely small positive quantity; e_α is the charge of a particle of species α; f_α is the distribution function, normalized to N_α, the number of particles per unit volume.

As is usually done, let us expand the distribution function sought in Sonine-Laguerre polynomials*

$$f_\alpha = f_{\alpha 0}\left(1 + \sum_{r=0}^{\infty}(\mathbf{v}_\alpha \mathbf{C}_{r,\alpha})L_r^{3/2}\left(\frac{m_\alpha v_\alpha^2}{2\chi T_\alpha}\right)\right). \tag{5}$$

Then for the coefficients $C_{r,e}$ we obtain the following system of equations:

$$\delta_{s0}\left[-\frac{\partial \ln N_{eT} e}{\partial r} + \frac{eE}{\chi T_e}\right] + \frac{5}{2}\frac{\partial \ln T_e}{\partial r}\delta_{s,1} = \sum_r \nu_0\{M_{sr} + \delta M_{sr}\}C_{r,e}, \tag{6}$$

$$\nu_0 = \frac{4}{3}\frac{N_e e^4\sqrt{2\pi}}{(\chi T_e)^{3/2}\sqrt{m_e}}, \tag{7}$$

$$\begin{aligned} M_{sr} = {} & \left|\frac{e_i}{e}\right| \ln(k_{\max}^{ei} r_D)\begin{pmatrix} 1 & 3/2 & 15/4 & 105/8 & \cdots \\ 3/2 & 13/4 & 69/8 & 495/16 & \cdots \\ 15/4 & 69/8 & 433/16 & 3231/32 & \cdots \\ 105/8 & 495/16 & 3231/32 & 26613/64 & \cdots \end{pmatrix} \\ & + \sqrt{2}\ln(k_{\max}^{ee} r_D)\begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 3/2 & 45/16 & \cdots \\ 0 & 3/2 & 45/4 & 927/32 & \cdots \\ 0 & 45/16 & 927/32 & 57825/256 & \cdots \\ . & . & . & . & \cdots \end{pmatrix}; \end{aligned} \tag{8}$$

$$\delta M_{sr} = -\frac{1}{\sqrt{2\pi}} \int_0^\infty dt\, e^{-t^2} N_{sr}(t^2) \left[\frac{1}{2}\ln(A^2+B^2) + \frac{A}{B}\left(\frac{\pi}{2} - \operatorname{arc\,tg}\frac{A}{B}\right)\right]. \quad (9)$$

Here $k_{\max}^{\alpha\beta}$ is the maximum wave number transferred in a collision, determined, as usual, either by the inapplicability of the classical treatment or by the inapplicability of perturbation theory; r_D is the Debye radius $\left(r_D^{-2} = \sum_\alpha 4\pi e_\alpha^2 N_\alpha/\chi T_\alpha\right)$. In formula (9)

$$N_{sr}(x) = \begin{pmatrix} 0 & 0 & 0 & \cdots \\ 0 & 1+2x & 3+3x-2x^2 & \cdots \\ 0 & 3+3x-2x^2 & 13+16x-3x^2+2x^3 & \cdots \\ . & . & . & . \end{pmatrix}, \quad (10)$$

and the functions A and B have the form

$$A = 1 - \frac{1}{1+\left|\frac{e_i}{e}\right|\frac{T_e}{T_i}} \left\{\operatorname{Re} J_+(t) + \left|\frac{e_i}{e}\right|\frac{T_e}{T_i}\operatorname{Re} J_+\left(t\sqrt{\frac{M_{iT}e}{m_{eT}i}}\right)\right\}; \quad (11)$$

$$B = \frac{\sqrt{\pi/2}\,t}{1+\left|\frac{e_i}{e}\right|\frac{T_e}{T_i}} \left\{e^{-t^2/2} + \left|\frac{e_i}{e}\right|\sqrt{\frac{M_i}{m_e}}\left(\frac{T_e}{T_i}\right)^{3/2} e^{-M_{iT}et^2/2m_{eT}i}\right\}, \quad (12)$$

where

$$J_+(t) = te^{-t^2/2}\int_{+i\infty}^t d\tau\, e^{\tau^2/2}.$$

The principal contribution to the integral (9) is made by the region of small values of t, in which, on the one hand, $A(t) < 0$, which corresponds to the possibility of the existence of re-

---

*

$$\int_0^\infty dx\, e^{-x} x^{3/2} L_r^{3/2}(x) L_{r'}^{3/2}(x) = \delta_{r,r'}\Gamma(r+5/2).$$

of the dispersion equation of plasma oscillations corresponding to sound waves, and, on the other hand, $B(t) \ll 1$, which corresponds to sufficiently weak absorption of such waves. It is easy to see that $A < 0$ for $t^2 < |e_i/e|(M_i^{-1}m_e) \ll 1$. Therefore

$$\delta M_{sr} \cong N_{sr}(0)\, I, \tag{13}$$

where

$$N_{sr}(0) = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 3 & {}^{45}/_{4} & \cdots \\ 0 & 3 & 13 & {}^{207}/_{4} & \cdots \\ 0 & {}^{45}/_{4} & {}^{207}/_{4} & {}^{3897}/_{16} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix},$$

$$I = -\frac{1}{\sqrt{2\pi}} \int_0^\infty dt\, e^{-t^2} \left[\frac{1}{2} \ln(A^2 + B^2) + \frac{A}{B} \left(\frac{\pi}{2} - \operatorname{arc\,tg} \frac{A}{B} \right) \right]. \tag{14}$$

Let us note that $B(t)$ ceases to be small when $t^2 \sim (m_e/M_i)(T_i/T_e)$. Therefore, in order to obtain an asymptotic estimate of the integral I, suitable for sufficiently large values of the ratio (T_e/T_i), we shall take it within the limits from $(m_{eT} i/M_{iT} e)^{1/2}$ to $|e_i m_e/eM_i|^{1/2}$. In this case

$$A \cong \frac{T_i}{T_e} \left\{ \left| \frac{e}{e_i} \right| - \frac{m_e}{M_i} \frac{1}{t^2} \right\}. \tag{15}$$

Neglecting the contribution of the logarithm and replacing arc tg by $(-\pi/2)$, we obtain

$$I \cong \frac{1}{2} \left| \frac{e_i}{e} \right| \frac{T_e}{T_i} \frac{1}{\ln \left[\dfrac{e_i^2 M_{iT} e^3}{e^2 m_{eT} i^3} \right]}. \tag{16}$$

This expression is the more accurate the larger it is in comparison with unity. The latter, in particular, is confirmed by comparing calculations by formula (16) with the results of numerical integration for $|e_i/e| = 1$ and $M_i/m_e = 1840$ (see Table 1).

Table 1

T_e/T_i	10	10^2	10^3	10^4	10^5	10^6
I numerical integr.	0.6	2.56	18.5	138	1180	10^4
I by (16)	0.35	2.35	17.8	142	1190	$1.02 \cdot 10^4$

For sufficiently large values of T_e/T_i, the matrix elements δM_{sr}, if they are not identically equal to zero, become the largest. Since in real plasmas the Coulomb logarithms may reach two tens, one may speak of sufficiently large values for $(T_e/T_i) > 10^3$ and not too large $|e_i/e|$. Under these conditions, for the density of the electric current $\mathbf{j}$ and the heat flux $\mathbf{q}$ due to electrons, the following expressions are obtained:

$$\mathbf{j} = \frac{eN_e\chi T_e}{m_e}\left\{\frac{1}{\nu_{\text{eff}}}\left[\frac{e\mathbf{E}}{\chi T_e} - \frac{\partial \ln N_{eT}e}{\partial \mathbf{r}}\right] - 5.1\,\frac{1}{I\nu_0}\frac{\partial \ln T_e}{\partial \mathbf{r}}\right\}, \tag{17}$$

$$\mathbf{q} = \frac{N_e(\chi T_e)^2}{m_e I\nu_0}\left\{5.1\left[\frac{e\mathbf{E}}{\chi T_e} - \frac{\partial \ln N_{eT}e}{\partial \mathbf{r}}\right] - 21.2\,\frac{\partial \ln T_e}{\partial \mathbf{r}}\right\}, \tag{18}$$

where

$$\nu_{\text{eff}} = \frac{4}{3}\,\frac{e^2e_i^2N_i\sqrt{2\pi}}{(\chi T_e)^{3/2}\sqrt{m_e}}\,\ln(k^{ei}_{\max}r_D). \tag{19}$$

To determine the viscosity, the nonequilibrium addition to the distribution function must, as is known, be expanded in the polynomials $L_r^{5/2}$:

$$\delta f = f_0\left(\frac{\partial V_i^0}{\partial x_k} + \frac{\partial V_k^0}{\partial x_i} - \frac{2}{3}\delta_{ik}\,\text{div}\,\mathbf{V}_0\right), \quad \left(v_iv_k - \delta_{ik}\frac{1}{3}v^2\right)\sum_r D_rL_r^{5/2}\left(\frac{m_ev^2}{2\varkappa T_e}\right). \tag{20}$$

In what follows we shall restrict ourselves to considering only that temperature region in which the principal role is played by the interaction of electrons with electrons, more precisely, the interaction due to plasma waves. Then we have the following system of equations for the coefficients D_r:

$$\delta_{s0} = -\frac{12}{5}\frac{\varkappa T_e}{m_e}\nu_0 I\sum_r P_{sr}D_r, \tag{21}$$

where

$$P_{sr} = \begin{pmatrix} 1 & 3/2 & 15/4 & \cdot \\ 3/2 & 17/4 & 93/8 & \cdot \\ 15/4 & 93/8 & 705/16 & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}. \tag{22}$$

As a result, for the coefficient of viscosity we obtain the expression

$$\eta = 1.81\,\frac{\varkappa T_e^i}{\nu_0 I}\,N_e. \tag{23}$$

According to formulas (17), (18), and (23), the only reminder of the usual theory of the kinetic coefficients of a plasma is the expression for the conductivity (and diffusion), which corresponds to the Lorentz model. The latter is connected with the suppression, for $T_e \gg T_i$, of higher approximations in the calculation of the conductivity with the aid of an expansion in Sonine–Laguerre polynomials. In all the remaining coefficients the interaction with ions proves insignificant, and the determining factor is the interaction between electrons, or, in other words, the interaction with plasma oscillations.

Let us note that at very large values of the temperature ratio ($T_e/T_i > 10^4$) it becomes necessary to take into account the influence of polarization on collisions of electrons with ions. Such allowance leads to the fact that in formula (19) the term

$$K = \frac{T_e}{T_i} \frac{1}{\left\{\ln\left[\frac{e_i^2}{e^2}\frac{M_i}{m_e}\left(\frac{T_e}{T_i}\right)^3\right]\right\}^2}.$$

is added to the Coulomb logarithm.

The function K is much smaller than I, owing to the presence in the denominator of an extra power of the logarithm.

In conclusion, we indicate that at large (T_e/T_i), for the complex dielectric permittivity one may use an interpolation expression suitable at both large and small ω:

$$\varepsilon(\omega) = 1 - \frac{\omega_{Le}^2}{\omega(\omega + i\nu_{\text{eff}})},$$

and for a plasma in a magnetic field, under the assumption that the field does not affect the collisions, there is no distinction between the transverse and longitudinal conductivities. These results correspond to the Lorentz model of a plasma.

Lebedev Physical Institute
Academy of Sciences of the USSR

Received
2 II 1962

CITED LITERATURE

1. V. P. Silin, A. A. Rukhadze, *Electromagnetic Properties of Plasmas and Plasma-like Media*, 1961.

2. V. P. Silin, *JETP*, **40**, 1768 (1961).