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Abstract

Full Text

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On the Question of a Possible Change in the Gravitational Constant

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P. Dirac came to the conclusion that the gravitational constant G, entering into the laws of gravitation of Newton and Einstein, is in fact not constant, but may change with time ([1,2]). According to Dirac, $G \sim 1/T$. It seems to us that Dirac is right only in the sense of indicating the variability of the gravitational constant with time, and is mistaken regarding the law of its change.

Dirac justified his conclusions by the fact that if the pulsation frequency of elementary particles $\omega = 10^{24}$ sec^{-1} is taken as the unit of time, then the age of matter in our region of the universe, which is of the order $\tau = 10^{17}$ sec, in this dimensionless time scale, since the world time $T = \tau\omega$, will have a value of the order of 10^{41}. Dirac assumed that, since the ratio of electromagnetic forces to gravitational forces for elementary particles at the present time is also of the order of 10^{41}, then with the passage of time it was precisely the gravitational constant that decreased (at $T = 1$ this ratio was of the order of 1), while the mass of the particles and their charge remained constant. At the same time, Dirac himself expressed doubt concerning the fulfillment of the conservation laws under such a conception.

One should go somewhat further than Dirac and assume that not only the gravitational constant, but also other so-called constants may change with time.

Using the laws of conservation of energy and of the total charge (in absolute value) in the universe, or, more precisely speaking, in a subregion of the universe with radius of curvature 10^{29} cm, and also the empirical dependence between the number of particles N in this subregion of the universe and the dimensionless world time T, one may arrive at the relations

$$Nmc^2 = Mc^2 = E = \text{const}; \qquad Ne^2 = \text{const}; \qquad N = T^2,$$

where e is the charge, m is the mass of an elementary particle (nucleon), and M is the total mass in the given subregion of the universe.

We then find that

$$m \sim T^{-2}; \qquad \hbar \sim T^{-2}; \qquad e \sim T^{-1}; \qquad G \sim T;$$

$$R^* = cT; \qquad \alpha^* = \frac{e^2}{\hbar c} = \frac{1}{137} = \text{const}; \qquad r_0 = \text{const};$$

$$\omega = \text{const}; \qquad c = \text{const}; \qquad \frac{GM^2}{e^2} \sim T^{-1},$$

where α^* is the fine-structure constant; r_0 is the radius of an elementary particle (nucleon); R^* is the radius of curvature of space.

At the same time, Eddington' s result concerning the number of particles in our subregion of the universe is somewhat refined; namely, putting $T = 10^{41}$, we find that $N = 10^{82}$ (Eddington had $N = 10^{79}$). $M = 10^{58}$ g.

The scheme we are considering is of cosmological interest. With complete fulfillment of these conservation laws we obtain results,

to some extent in agreement with what is observed in reality. A mutual connection is established among various so-called cosmic and nuclear constants.

For $T = 1$, $R^* = r_0$, the gravitational radius $r_g = GM/c^2$ and the characteristic length determining the region of gravitational fluctuations (of the metric), $l_g = (G\hbar/c^3)^{1/2}$, also were equal to r_0.

Essential in the concept under consideration is the idea of the emission of gravitational waves by elementary particles (4-7). This radiation, caused by their mechanical quadrupole oscillations, leads to a loss of energy $E = \alpha E$, where $\alpha \simeq 10^{-17}\,\text{sec}^{-1} \sim 1/T$, i.e., it is quite substantial and leads to a decrease in the energy (rest mass) of elementary particles.

If we now consider the basic equation of the general theory of relativity, relating the curvature tensor to the energy-momentum tensor, and take the covariant derivatives of the two sides of this equation, then it is not difficult to see that both the left-hand and right-hand sides, for constant G and c, although they satisfy the Lorentz transformation group (in the small), nevertheless in a number of concrete cases, for example in the case of an "expanding" universe, do not satisfy the principle of similarity (the scale group) under a change of time.

Indeed, since

$$R_i^k - \frac{1}{2}\delta_i^k R = \frac{8\pi G}{c^4} T_i^k = \chi T_i^k, \tag{1}$$

then under a change of time, taking $c = \text{const}$, we have for the left-hand side a change of the radius of curvature $\sim T^{-2}$, whereas on the right-hand side, for constant χ, the energy density or pressure must vary as T^{-3} (and as T^{-4} for small T in the ultrarelativistic case). It follows from this that the quantity χ must vary with time, namely $\chi \sim T$. Thus, in order that in equation (1) the quantities on the right and on the left change with time according to the same law, one must require that the gravitational constant increase proportionally to time, i.e., we arrive at the assertion made above.

The principal shortcoming of the general theory of relativity is the not entirely correct fulfillment of the laws of conservation of energy-momentum on large scales; namely, upon covariant differentiation of the terms of equation (1), for constant χ we shall have

$$\left(R_i^k - \frac{1}{2}\delta_i^k R\right)_{;k} \equiv \chi T_{i;k}^k = 0.$$

However, the equation $T_{i;k}^k = 0$ does not express the laws of conservation of energy-momentum, since the energy-momentum tensor of matter T_i^k does not take into account the gravitational field generated by this matter.

To correct the equation $T_{i;k}^k$, A. Einstein introduced into it the so-called energy-momentum pseudotensor of the gravitational field t_i^k, varying the pseudoscalar $\mathfrak{G} = g^{ik}(\Gamma_{il}^m\Gamma_{km}^l - \Gamma_{ik}^l\Gamma_{lm}^m)$. According to Landau (3), the pseudotensor t_i^k is defined so that the condition

$$\frac{\partial}{\partial x^k}\left[(-g)(T^{ik} + t^{ik})\right] = 0,$$

is satisfied, where

$$2\chi(-g)\left[T^{ik} + t^{ik}\right] = \frac{\partial^2}{\partial x^l \partial x^m}(-g)\left[g^{ik}g^{lm} - g^{il}g^{km}\right],$$

which determines the pseudotensor t_i^k itself.

The nonconservation of the energy conservation law for a constant value of the gravitational constant is not accidental. One may suppose that the equations of mathematical physics express the laws of conservation of energy only when,

when all the terms of these equations transform according to one and the same group of transformations, which is not the case for equation (1) when χ is constant.

In the case of weak gravitational fields, when the basic equation of the theory of gravitation (1) can be considered in the linear approximation, the group properties of the left- and right-hand sides of this equation are the same, since linear additions to the curvature and energy-momentum tensors are considered, which for $M = \text{const}$, $G = \text{const}$, and $c = \text{const}$ have identical dimensions. In this case the pseudotensor t_i^k behaves as a tensor and, consequently, has a real meaning, and the conservation laws are exactly fulfilled. In the case of strong fields, as we saw above, this is no longer always the case, and one has to assume that $G \neq \text{const}$; this makes it possible in a natural way to generalize Einstein' s equation (1). Taking $\chi = \chi(g_{ik}x^i x^k)$ and varying $R^* = \frac{1}{2c\chi}\,g^{ik}R_{ik} = \frac{R}{2c\chi}$, equation (1) can be written in the form

$$R_i^k - {}^1\!/_2\delta_i^k R = \chi\,(T_i^k + t_i^k) - \delta_i^k R\,\frac{\partial \ln \chi}{\partial \ln g}, \tag{2}$$

where t_i^k is the tensor or pseudotensor of the field. Then, under covariant differentiation of (2), we shall have

$$\left(R_i^k - {}^1\!/_2\delta_i^k R\right)_{;k} = \{\chi(T_i^k + t_i^k)\}_{;k} - \frac{\partial \xi}{\partial x^i} = 0, \tag{3}$$

where $\xi = R\dfrac{\partial \ln \chi}{\partial \ln g}$.

Differentiation of the right-hand side gives

$$\frac{\chi}{\sqrt{-g}}\,\frac{\partial \sqrt{-g}\,(T_i^k + t_i^k)}{\partial x^k} - \frac{\chi}{2}\,(T^{kl} + t^{kl})\,\frac{\partial g_{kl}}{\partial x^i} + \frac{\partial \chi}{\partial x^k}(T_i^k + t_i^k) = \frac{\partial \xi}{\partial x^i}. \tag{4}$$

It may be assumed that the radiation of gravitational waves and, in general, the gravitational field are due to the fact that matter and, in particular, elementary particles, in creating the field, expend on this part of their energy (4-7). In this case the energy of the particles and the so-called world constants associated with these particles change with time and, apparently, may in general depend on all coordinates.

A generalization of this proposition may be the writing of these "constants" in the form of tensors, in particular of second rank, and the use, instead of Einstein' s equation (1), of an equation of the form

$$f(R^m_{ikl}) = A_l^m(T_{ik} + t_{ik}) + F\left(\frac{\partial A_l^m}{\partial \ln g};\ R_i^k\right),$$

where $f(R^m_{ikl})$ should be understood as a quasilinear combination of the unit and metric curvature tensors, satisfying the condition $f_{;k} = 0$. For $l = m$ we arrive at Einstein' s equation.

One may also investigate an equation more general than (2), when

$$R_i^k - {}^1\!/_2\delta_i^k R = A_i^r(T_r^k + t_r^k) + A_i^k R_l^m \frac{\partial B_m^l}{\partial \ln g} + \frac{1}{2}\left(A_i^k R_r^l B_l^r - \delta_i^k R\right),$$

where $A_i^r B_r^k = \delta_i^k$, and A_i^r is a tensor, or pseudotensor, generalizing the "gravitational constant." Let us note that, since the world constants are interrelated, then, assuming that the "gravitational constant" has tensor properties, we shall arrive at the conclusion that Planck' s "constant" and the generalized "mass" and "charge" also have tensor properties.

On the basis of the foregoing, the following possible (already discussed) model of the formation of visible objects in our Uni-

···finite. Some time ago (about 10 billion years), two ultrarelativistic particles collided, after which the multiple birth of new particles began; this continues even now, and the particles themselves age, while their number increases. At the same time, from the gravitational particles—gravitons—that appeared earlier, pairs of new particles now arise ([8]). Since M = const, the amount of matter arising corresponds to the amount of matter that is transformed into gravitational radiation. This amount is evidently determined by the relation $\dot{m} = \alpha M$, where $\alpha = 1/T \simeq 3 \cdot 10^{-18} - 10^{-18}$ sec^{-1}, which corresponds to the birth of matter of order 10^{-45} g in 1 cm^3 in 1 sec.

Thus, previously developed views on the continuous "creation" of matter are confirmed, but not from nothing; rather, from the gravitational field, which is close to Jordan' s views.

The process of "creation" and the gravitational interactions of all particles can be described by the general theory of relativity, but taking into account only the fact that the gravitational "constant" increases with time. "Our universe" may be a certain strongly nonstationary, non-Euclidean metric formation in an infinite universe, whose metric as a whole may be closer to Euclidean.

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Note: Figure translations are in progress. See original paper for figures.