# A. M. PERELOMOV

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## Abstract

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# A. M. PERELOMOV

## POSSIBLE DETERMINATION OF ADDITIONAL CHARACTERISTICS OF AN UNSTABLE PARTICLE

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At the present time there is no complete theory of unstable particles. Various authors ([1−4]) define the concept of an unstable particle in different ways; however, all of them note the difficulties that arise in doing so. These difficulties can to a large extent be overcome if one assumes that at the beginning and at the end of the process we are dealing with stable particles, while unstable particles appear only virtually and serve merely as a convenient way of describing resonances in scattering processes and particle transformations. From this point of view, the decay process of an unstable particle $C^* \to C + D$ cannot, strictly speaking, be separated from the process of its formation; rather, one should regard it as a single process, for example $A + B \to C^* + E \to C + D + E$, where the particles $A, B, C, D, E$ are stable, and the spectrum of $E$ has a resonance at the energy $E_A + E_B - m_{C^*}$ ([5,6]). (Of course, in a number of cases one may approximately assume that the decay of the particle does not depend on the way in which it is formed.)

In this note we wish to draw attention to the fact that, in addition to the characteristics that are usually considered (the mass of the unstable particle and its lifetime), one can "determine" other characteristics of an unstable particle as well, for example its magnetic dipole moment and electric dipole moment. The magnetic dipole moment may manifest itself in the scattering of particles polarized along the $z$-axis in a magnetic field directed along the $x$-axis: after scattering, the polarization will be rotated in the $zy$ plane. For lack of space the calculation is not given. Let us note only a curious fact. The change in the mean value of the spin, calculated with the complete wave function, i.e., with the incident plus the scattered wave, is not equal to the mean value of the spin for the scattered wave alone. This means that the spins of the transmitted particles undergo a rotation. The spin rotation of the scattered particles turns out to be proportional to $\sim [\gamma^2/((E-\varepsilon)^2+\gamma^2)]^2$, while the total rotation of the scattered and transmitted particles is $\sim \gamma^2/((E-\varepsilon)^2+\gamma^2)$.

In this note we consider the scattering of neutral stable particles in an electric field through an intermediate state interacting with virtual charged fields. It turns out that in the stationary case there is no spin rotation. In the scattering of a wave packet, however, a spin rotation of neutral stable particles arises, but such that the time average of the rotation is zero, while the first moment of the rotation with respect to time is nonzero.

This result is obtained in a theory invariant under time inversion but not conserving spatial parity; the spin rotation corresponds to the notion of an electric dipole moment (e.d.m.) of the unstable state ([7]). In the case of a stable particle, as is known ([8]), the e.d.m. is absent.

Let us consider the model proposed by Ya. B. Zel' dovich with five particles $a, b, c, d$, and $e$. Let particles $a$ and $e$ be bosons, $b, c$, and $d$ fermions; $a, b$, and $c$ are neutral, $d$ is positively charged, and $e$ negatively charged. We choose the interaction so that particle $c$ can virtually decay into particles $d$ and $e$, and so that the decay occurs with nonconservation of parity. Let $c$ also decay really into $a$ and $b$ ($m_a+m_b < m_c < m_d+m_e$). We place the system in a homogeneous electric field directed along the $z$-axis.

Such a system is described by the Hamiltonian

$$\begin{aligned}\mathcal{H} = &\sum_{x=a,b...} \int \psi_x^+(\mathbf{r}) \left(m_x + \frac{p^2}{2m_x}\right) \psi_x(\mathbf{r})\, d\mathbf{r} + \\ &+ \sqrt{4\pi}\, f \int \psi_c^+(\mathbf{r})\psi_b^+(\mathbf{r})\psi_a(\mathbf{r})\, d\mathbf{r} + \sqrt{4\pi}\, f^* \int \psi_a^+(\mathbf{r})\psi_b^+(\mathbf{r})\psi_c(\mathbf{r})\, d\mathbf{r} + \\ &+ \sqrt{4\pi}\, g \int \psi_c^+(\mathbf{r})\psi_d(\mathbf{r})\psi_e(\mathbf{r})\, d\mathbf{r} + \sqrt{4\pi}\, g^* \int \psi_e^+(\mathbf{r})\psi_d^+(\mathbf{r})\psi_c(\mathbf{r})\, d\mathbf{r} + \\ &+ \sqrt{4\pi}\, ih \int \psi_c^+(\mathbf{r})\, \vec{\sigma}\, \psi_d(\mathbf{r})\, \frac{d\psi_e(\mathbf{r})}{dr} - \sqrt{4\pi}\, ih^* \int \frac{d\psi_e^+}{dr}\, \psi_d^+ \vec{\sigma} \psi_c\, d\mathbf{r} + \\ &+ F \int \psi_d^+(\mathbf{r})\, z\psi_d(\mathbf{r})\, d\mathbf{r} - F \int \psi_e^+(\mathbf{r})\, z\psi_e(\mathbf{r})\, d\mathbf{r}.\end{aligned} \tag{1}$$

In a theory invariant with respect to time reversal, $f^* = f$, $g^* = g$, $h^* = h$. The state vector has the form

$$\begin{aligned}\Phi = \Big[\int \psi_a^+(\mathbf{r}_1)\psi_b^+(\mathbf{r}_2)\psi(\mathbf{r}_1\mathbf{r}_2)\, d\mathbf{r}_1\, d\mathbf{r}_2 + \int \psi_c^+(\mathbf{r})\varphi(\mathbf{r})\, d\mathbf{r} + \\ + \int \psi_d^+(\mathbf{r}_1)\psi_e^+(\mathbf{r}_1)\chi(\mathbf{r}_1\mathbf{r}_2)\, d\mathbf{r}_1\, d\mathbf{r}_2\Big] |0\rangle,\end{aligned} \tag{2}$$

and in the center-of-inertia system the equation $\mathcal{H}\Phi = E\Phi$ can be written as follows:

$$E\psi(\mathbf{r}) = \left(m_a + m_b - \frac{\Delta}{2\mu_{ab}}\right) \psi(\mathbf{r}) + \sqrt{4\pi}\, f\delta(\mathbf{r})\varphi; \tag{3}$$

$$E\chi(\mathbf{r}) = \left(m_d + m_e - \frac{\Delta}{2\mu_{de}}\right) \chi(\mathbf{r}) + Fz\chi(\mathbf{r}) + \sqrt{4\pi}(g + ih\vec{\sigma}\nabla)\delta(\mathbf{r})\varphi, \tag{4}$$

$$E\varphi = \mu\varphi + \sqrt{4\pi}(g + ih\vec{\sigma}\nabla)\chi(\rho) + \sqrt{4\pi}f\psi(\rho) \tag{5}$$

$$\mu_{ab} = m_a m_b/(m_a + m_b); \quad \mu_{de} = m_d m_e/(m_e + m_d); \quad 2\mu_{de}(m_d + m_e - E) = \varkappa^2;$$

$$2\mu_{ab}(E - m_a - m_b) = k^2; \quad 2\mu_{de}(m_d + m_e - \varepsilon) = \varkappa_0^2; \quad 2\mu_{ab}(\varepsilon - m_a - m_b) = k_0^2;$$

$\mu$ is the unrenormalized mass of particle $c$; $\rho$ is the cutoff radius, which after renormalization is made to tend to zero (see the appendix to paper ([9])).

In the first approximation we neglect $Fz$. We find an exponential solution with complex energy $E_0 = \varepsilon - i\gamma$. The real and imaginary parts of $E_0$ determine, respectively, the mass of particle $c$ and the probability of its decay. To simplify the formulas we shall regard the parity-nonconservation constant and the decay constant as small and neglect terms with $h^2$, $f^4$, etc. In addition, we assume $\gamma \ll m_d + m_e - \varepsilon$, $\gamma \ll \varepsilon - m_a - m_b$, which leads to the restrictions $2\mu_{ab}f^2 \ll \varkappa_0^2/2\mu_{de}k_0$, $2\mu_{ab}f^2 \ll k_0/2\mu_{ab}$. We obtain

$$\mu = \varepsilon + 2\mu_{de}g^2\frac{1}{\rho} + 2\mu_{ab}f^2\frac{1}{\rho} - 2\mu_{de}g^2\varkappa_0; \tag{6}$$

$$\gamma = \frac{2\mu_{ab}f^2k_0}{1 + 2\mu_{de}g^2\mu_{de}/\varkappa_0}. \tag{7}$$

We note that if the second term in the denominator is $\gg 1$, then a narrowing of the resonance occurs owing to the strong coupling of $e, d$ and $c$,

$$\gamma \simeq \frac{\mu_{ab}f^2}{\mu_{de}g^2}\,\frac{k_0\varkappa_0}{\mu_{de}}.$$

If particle $c$ can transform into other particles $d'$ and $e'$, $d''$ and $e''$, etc., this leads to the addition of new positive terms to the denominator and to an even greater narrowing of the resonance. If this model could satisfactorily describe unstable particles, then the narrow resonances recently observed experimentally could be explained by such a mechanism. This idea was independently expressed by B. L. Ioffe.

We now take $Fz$ into account in first order. Determining $\psi$ and $\chi$ from equations (3) and (4) and substituting into equation (5), we find that there are no corrections to $E_0$ linear in $F$. In this case the expression for $\delta E_0$ is the same as in perturbation theory for complex $E_0$ ([10,11]): $\delta E_0 \simeq F\int \psi_0^* z\psi_0\, d\mathbf{r} = 0$. Thus, the circumstance that $\delta E_0 = 0$ means, at first glance, a violation of Newton' s third law (action equals reaction). Indeed, an unstable particle creates a dipole electric field, since $\int \psi_0^* z \psi_0 \, dr \neq 0$, while a homogeneous electric field does not cause precession of its spin about the direction of the field, since there is no splitting, $\delta E_0 = 0$. However, this contradiction is only apparent and means only that unstable particles cannot be treated in the same way as stable ones. If, however, we assume that at the beginning and at the end of the process we have stable particles, i.e., if we do not separate the process of formation from the process of decay, then no such paradox arises. The precession effect may appear in the case of scattering of particles $a$ and $b$. If, before scattering, particle $b$ is polarized along the $z$-axis, and the electric field is directed along the $x$-axis, then one may expect that after scattering its polarization is rotated in the $yz$ plane through some angle $\theta$. In order for this effect to occur, it is also necessary that the state of the system $a, b$ not be a state with a definite energy, i.e., it is necessary to consider the scattering of wave packets. (The effect of an electric dipole moment in scattering is an effect of the type $[\vec{\sigma}\,\vec{\sigma}\,']\,F$ and, by virtue of invariance with respect to combined inversion, disappears if the system $a, b$ has a definite energy.) Let us note once again that in the case of a magnetic moment, even in the stationary case there is already a rotation of the spin.

Our problem reduces to solving the system of equations

$$i\frac{d\psi}{dt} = \left(m_a + m_b - \frac{\Delta}{2\mu_{ab}}\right)\psi + \sqrt{4\pi f}\,\delta(r)\varphi; \tag{3'}$$

$$i\frac{d\chi}{dt} = \left(m_d + m_e - \frac{\Delta}{2\mu_{de}}\right)\chi + Fr\chi + (g + ih\vec{\sigma}\nabla)\,\delta(r)\varphi; \tag{4'}$$

$$i\frac{d\varphi}{dt} = \mu\varphi + (g + ih\vec{\sigma}\nabla)\,\chi(\rho) + f\psi(\rho). \tag{5'}$$

We choose the following initial conditions:

$$\psi(r,0) = \psi_0(r)\alpha, \qquad \chi(r,0) = 0, \qquad \varphi(r,0) = 0;$$

$$\psi_0 = \frac{1}{2\sqrt{\pi}\sqrt{\pi a}}\, e^{-ipr-(r-r_0)^2/2a^2}. \tag{8}$$

Let us calculate

$$\frac{ds(t)}{dt} = \frac{d}{dt}\left[\int \psi^* \frac{\vec{\sigma}}{2}\psi\, dr + \int \chi^* \left(\frac{\vec{\sigma}}{2} + [rp]\right)\chi\, dr + \varphi^* \frac{\vec{\sigma}}{2}\varphi\right]. \tag{9}$$

Using $(3')$, $(4')$, and (5), we obtain

$$\frac{ds(t)}{dt} = \int \chi^*(r,t)\,[Fr]\,\chi(r,t)\,dr. \qquad (10)$$

We integrate this equality with respect to time from 0 to $\infty$.

$$\Delta s = \int \chi^*(r,t)\,[Fr]\,\chi(r,t)\,dr\,dt. \qquad (11)$$

To calculate $\Delta s$, we shall use the fact that the functions

$$\psi_k = \frac{2}{2\pi\sqrt{2}}\left(\frac{-e^{-ikr} + S(k)e^{ikr}}{r}\right); \qquad (12)$$

$$\chi_k = \frac{1}{2\pi\sqrt{2}}\,\frac{\mu_{de}}{\mu_{ab}}\,(S(k)-1)\,\frac{(g + ih\vec{\sigma}\nabla)}{f}\,\frac{e^{-\chi r}}{r}; \qquad (13)$$

$$\varphi_k = -\frac{S(k)-1}{2\sqrt{2\pi\mu_{ab}}\,f} \qquad (14)$$

form a complete orthonormal system. Here

$$S(k) = \frac{(\varepsilon - E)\left[1 + 2\mu_{de}g^2\frac{2\mu_{de}}{\chi+\chi_0}\right] + i2\mu_{de}f^2k}{(\varepsilon - E)\left[1 + 2\mu_{de}g^2\frac{2\mu_{de}}{\chi+\chi_0}\right] - i2\mu_{de}f^2k}. \qquad (15)$$

(the formulas are given for $E < m_d + m_e$). For $E > m_d + m_e$, $\chi$ is replaced by $-i\sqrt{2\mu_{de}(E - m_d - m_e)}$. Expanding the functions $\psi_0, \psi, \chi, \varphi$ in the complete system, we find, neglecting exponentially small terms, $\Delta s = 0$.

Let us now calculate the first moment of the spin rotation, i.e., compute the integral

$$\bar{t} = \int_0^\infty \frac{ds(t)}{dt}\,t\,dt.$$

Omitting the calculations, we give the result:

$$\bar{t} = -\frac{F}{3}\,\frac{\mu_{de}^3}{\mu_{ab}^2}\,\frac{gh}{f^2}\,\frac{1}{(2\chi)^3}\,\frac{\mu_{ab}}{k}\,|S(k)-1|^2. \qquad (16)$$

On the other hand, knowing the exponential solution describing the unstable particle, it is not difficult to find

$$d = \int \psi_{0z}^* \psi_0 \, dr = gh \frac{2\mu_{de}^3 \gamma}{3\chi_0^3} \frac{1}{1 + 2\mu_{de} g^2 \dfrac{\mu_{de}}{\chi_0}}. \tag{17}$$

Using formula (7), we obtain

$$\bar{t} = -\tau(\tau F d) \frac{\chi_0^3}{\chi^3} \frac{k_0}{k} \frac{1}{4} |S(k) - 1|^2, \qquad \tau = \frac{1}{\gamma}. \tag{18}$$

Formula (17) corresponds to the dipole moment of an exponentially decaying particle. The dipole moment turns out to be proportional to $\gamma$, i.e., to the time derivative of the wave function. Taking this into account, we interpret (18) as follows: at the beginning of the scattering process, when the amplitude of the wave function grows, the dipole moment has one sign and the spin rotation occurs in one direction; at the end of the scattering process, the amplitude of the unstable particle decreases, the dipole moment has the other sign and the spin rotation occurs in the opposite direction. On average over time, the spin rotation turns out to be equal to zero. In this case the sign of the moment of the spin rotation, as one would expect, coincides with the sign of the rotation angle of the decaying particle. The structure of the formula obtained is also clear: $\bar{t}$ is proportional to the lifetime of the particle $\tau$, to the magnitude of the spin rotation of the unstable particle $(\tau F d)$, to the resonance factor $\frac{1}{4}|S(k)-1|^2$, equal to unity at resonance, and, finally, to a factor weakly dependent on energy, equal to unity at resonance.

From this one may draw the following conclusions: if it were possible to measure the polarization of the scattered particle during the scattering of a wave packet as a function of time (the time being counted from the moment of preparation of the wave packet), then one should expect that the polarization initially rotates in one direction and then in the other.

In conclusion, I express my gratitude to Academician Ya. B. Zel' dovich, who pointed out the formal violation of Newton's third law, for numerous discussions and valuable advice, and also to B. L. Ioffe, who took part in the discussion.

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*Note: Figure translations are in progress. See original paper for figures.*