Academician V. V. SHULEIKIN

1962

SovietRxiv

Abstract

Full Text

GEOPHYSICS

Academician V. V. SHULEIKIN

FURTHER ON THE BEHAVIOR OF A LIQUID LOSING WEIGHT

In the preceding article ([1]) we considered changes in the shape of the surface of water in a glass vessel under a gradual decrease of weight down to zero. It was shown why, in this process, the free surface of the water increases, and at the expense of what the surface energy grows—as occurred in the experiments of pilot-cosmonaut P. R. Popovich aboard the spacecraft *Vostok-4*.

We shall now trace quite different changes in the shape of the surface of water in contact with a hydrophobic wall and, in the end, assuming the form schematically shown in Fig. 2 in ([1]).

For definiteness, let us suppose that the underlying surface is covered with a layer of zinc stearate, with which the surface of water forms a particularly large contact angle: $\theta = 135^{\circ}$, according to ([2]). This is somewhat less than the contact angle of a mercury drop with glass.

Let us take the volume of water equal to one liter. In the initial state the water will lie on the plane in a thin layer, the edges of which satisfy the equation of the meridional section of a very large drop, well known from the literature. Let us use, for example, equation (2) from ([3]). Substituting into it the value of the tangent of the contact angle $p = -1$ and the factor before the bracket, which in the notation of ([1]) will be $h^2 = 3.86^2$; choosing a minus sign before the first term in the bracket, we then see that the thickness of the water layer is about 5 mm. Correspondingly, a liter of water may occupy the area of a circle of radius 250 mm. This "limitingly flat drop" will begin gradually to contract toward the axis passing through the center of the circle when, after the elimination of overloads, the gravitational and centrifugal field will be characterized by the resultant value $g_1 < g$, before the establishment of the spacecraft' s stationary motion in orbit.

The profile of the meridional section of drops of any size, in the most general form, is described by the equation of J. Adams ([4]), which we shall now use instead of the equations of A. N. Krylov ([5]) applied in ([1]). Let the origin of a cylindrical coordinate system be located at the apex of the drop, and let the coordinate z be measured downward. We denote the shortest distance of a point of the profile from the axis of symmetry by x, and the angle between the z-axis and the normal at the given point of the profile by φ. We denote by r_1 the radius of curvature of the meridional section at the point under study. Its largest value

b—at the apex of the drop—Adams takes as the scale length for transition to dimensionless quantities. In our notation this is written as follows: $x/b = \xi$; $z/b = \zeta$; $r_1/b = \rho_1$, after which Adams' s fundamental equation assumes the form:

$$\frac{1}{\rho_1} + \frac{\sin\varphi}{\xi} = 2 + \beta\zeta. \tag{1}$$

In turn, the important parameter β is expressed in terms of b and quantities characterizing the substance and the force field:

$$\beta = \frac{\delta g b^2}{\sigma}. \tag{2}$$

As in ([1]), here δ denotes the density of water, and σ the surface tension.

Just as system (6) in ([1]), equation (1) now describes whole families of curves. But in ([1]) the curves proved to be similar while preserving

constant ratio of the vessel radius to the scale length h; in the new problem, similarity of the curves is ensured when a constant value of β is maintained. Consequently, we can again dispense with integrating the differential equations, now following from (1), and even with calculating the coordinates from special tables, if we use the previously computed droplet profiles corresponding to various parameters β, and perform a simple recalculation of the coordinates on the basis of formula (2). Such profiles are given in the book by Bashforth and Adams ([4]), in Fig. 3, for various drops of mercury on glass. Let us consider four of them, denoted there by the numbers 4, 8, 16, 24. They correspond to drop weights of 0.260, 0.519, 1.037, and 1.55 g. The contact angle is taken to be 147°; hence, for our problem it is necessary to subtract from the volume of the drops those parts that lie below the points of the profiles where the tangent makes an angle of 135° with the supporting surface.

After this, the volumes V subject to investigation are reduced, respectively, to 18.3, 36.8, 74.6, and 113.2 mm^3.

We shall denote, by letters without subscripts, the quantities corresponding to the calculations of Bashforth and Adams (but with the volumes reduced as mentioned), and by the same letters with subscript 1, the corresponding quantities under the conditions studied: applied to water in the amount of 1 liter and to the decreasing value g_1.

Then the condition of similarity of the contours, on the basis of (2), is written:

$$\frac{g_1\delta_1 b_1^2}{\sigma_1} = \frac{g\delta b^2}{\sigma}. \tag{3}$$

On the other hand, the condition must be satisfied:

$$\left(\frac{b_1}{b}\right)^3 = \frac{V_1}{V}. \tag{4}$$

From (3) and (4) it follows that:

$$\frac{g}{g_1} = \frac{\delta_1}{\delta}\frac{\sigma}{\sigma_1}\left(\frac{V_1}{V}\right)^{2/3}. \tag{5}$$

By this many times must weightiness be reduced in order that a liter of water be enclosed within a surface that is similar in form to one of the mercury drops considered (without the layer subtracted below).

The increase of scale in comparison with the profiles of the mercury drops was calculated by us from (4); for V we substituted the four indicated values of the volume.

The results of the calculations belong to the category of strong reductions of weightiness. Therefore, for completeness of the series, we calculated one more profile—according to the table contained in ([4])—for the largest value of the parameter in the tables, $\beta = 100$.

Figure 1 presents the right halves of the meridional sections of the seven surfaces obtained. Curve a corresponds to the initial state; here both the contour itself and the trace of the supporting surface are broken so that they fit in the drawing. Curve b, calculated from the tables of Bashforth and Adams, and curves c–f, constructed on the basis of the law of similarity (3), (4), correspond to successive reductions of weightiness: by factors of 100, 200, 270, 430, and 700.

Circle k (with its center marked by a small circle on the ordinate axis), approaching the plane at the former angle of 135°, corresponds to complete weightlessness of water: here, at all points of the surface of the sphere enclosing within itself 1 liter of water, the same pressure is established, in accordance with equation (10) from (1). To judge the scale of the drawing, distances expressed in millimeters are marked along the axis of rotation and along the trace of the supporting surface.

The transition from the regime of overloads through the normal value g to values g_1 decreasing to zero occurs on spacecraft very rapidly. It is therefore of interest to determine how rapidly successive changes in the form of the liquid surface can proceed under the action of molecular forces and with the participation of inertial forces.

A sufficiently clear estimate of the dynamic aspect of the phenomena can be made according to a simple scheme: the liquid is initially in state a (Fig. 1); it instantaneously loses weight and freely tends toward state k.

In a state close to a, the potential (surface) energy of the system is approximately proportional to x^2, while the kinetic energy is approximately proportional to

Fig. 1

Figure 1: Fig. 1

$(dx/dt)^2$, where x is the distance from the axis to the edge of the "flat drop," and t is the current time. As the limiting form k is approached, conditions arise that are still more similar to those well known in the problem of oscillations of the shape of drops. In essence, freely falling drops are in a state of a peculiar "weightlessness" : the forces of gravity are completely balanced by the forces of inertia during the accelerated motion of the whole mass; oscillations of shape are governed only by surface tension and by inertial forces during the displacements of particles relative to the center of mass of the drop. As is known, in this case drops successively take the form of compressed ellipsoids of revolution, spheres, and elongated ellipsoids of revolution.

Fig. 1

In Fig. 2 the initial state of weightless water is shown, bounded above by the surface of a compressed ellipsoid of revolution A. The contact angle with the underlying surface is taken to be 90° (it differs little from the contact angle of water with the surface of paraffin). At the bottom of Fig. 2 a segment corresponding to 1 dm is presented; at this scale the volume of water is equal to 1 l. Let us disregard the energy concentrated at the water–solid interface. Then it turns out that surface tension creates on the surface A, equal to 5.56 dm^2, a potential energy of $4.06 \cdot 10^6$ erg. Having contracted into a sphere B, the water will acquire an external surface of 3.82 dm^2, to which an energy of $2.79 \cdot 10^6$ erg corresponds. In the absence of losses, here $1.27 \cdot 10^6$ erg has been transformed into the kinetic energy of the moving water masses; therefore the deformation of the bodies must continue until the elongated ellipsoid C obtains a surface equal to the initial 5.56 dm^2. After

this system will go back to state A, etc.; the oscillations would not die out if there were no energy losses.

The period of these oscillations does not differ from the period of oscillations of complete ellipsoids, since the picture of the motion of the particles in the absent lower half would be a simple mirror image and would not in any way affect the motions in the upper half. Into the classical formula derived by Rayleigh ([6]), let us substitute, instead of the mass δV, the doubled mass of the water $2m$. Then for the period T we obtain:

$$T = \sqrt{\frac{3}{4}\pi \frac{m}{\sigma_1}}. \tag{6}$$

If $m = 1000$ g and $\sigma_1 = 73$ dyn $\cdot$ cm^{-1}, then $T = 5.68$ sec.

Fig. 2

Fig. 2

Figure 2: Fig. 2

Unlike the problem of the mathematical pendulum, the correction to (6) for large amplitudes here should be insignificant. Consequently, under conditions of weightlessness, 1 l of water that was at rest in state a (Fig. 1) can assume a spherical form in 1.4 sec.

In reality, the phenomena must proceed in a more complicated way than in the scheme described. First of all, it can be shown that taking into account the energy at the water-solid interface—in the particular case of paraffin—will increase the potential energy in state A by approximately 60%, in state B by 34%, and in state C by 7%. This will lead to a decrease in the period of the oscillations by approximately 20%. In principle it is possible to use observations of such oscillations to determine the surface energy of those contacting liquid and solid bodies for which no experimental methods have so far been proposed. However, other aspects of the phenomenon are still unknown: nothing can be said about the damping of the oscillations between states A and C under the influence of friction against the underlying surface. Undoubtedly, the oscillations must be damped still more strongly under the influence of the slightest traces of surface-active substances in the water—for the same reason that surface-active substances damp small waves on the surface of water ([7]). Of course, the same surface-active substances will also change the period T: for example, when σ_1 is lowered to 30 dyn $\cdot$ cm^{-1}, the period of the oscillations should increase by 56% and, in the example considered, reach 8.8 sec.

Received
20 IX 1962

CITED LITERATURE

[1] V. V. Shuleikin, DAN, **147**, No. 1, 92 (1962).
[2] A. Nietz, J. Phys. Chem., **32**, 255 (1928).
[3] H. Geiger, K. Scheel, Handb. d. Phys., **7**, 362 (1927).
[4] F. Bashforth, J. C. Adams, *An Attempt to Test the Theories of Capillary Action*, 1883, p. 15.
[5] A. N. Krylov, Arkh. fizicheskikh nauk, **1**, issue 1-2, 110 (1918).
[6] J. Strutt (Lord Rayleigh), *Theory of Sound*, **2**, 362 (1955), 2nd Russian ed.
[7] V. V. Shuleikin, *A Short Course in the Physics of the Sea*, 1959, p. 434.

Note: Figure translations are in progress. See original paper for figures.