CYBERNETICS AND CONTROL THEORY

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Fig. 1

Figure 1: Fig. 1

Abstract

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CYBERNETICS AND CONTROL THEORY

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ON THE SYNTHESIS OF SYSTEMS WITH A RIGID STRUCTURE EQUIVALENT TO SELF-ADJUSTING SYSTEMS

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We shall consider controlled objects in which the regime that is optimal from the point of view of some quality criterion is disturbed: a) because of external disturbances that enter the object and whose influence leads to a change in the characteristics of the object, and b) when the characteristics of the object change during operation independently of external disturbances*. We shall consider the structure obtained to be equivalent to a self-adjusting system if, for case a), the processes in the system do not depend on external disturbances and, for case b), the Bode sensitivity $S^{K(P)}_{m_2(P)}$ ([1]) does not depend on the transfer function of the object. The question of choosing the quality criterion is not considered here; it is assumed that the specification for the control system corresponds to the chosen quality criterion and can be realized by one of the methods considered in ([2]). It is essential that such a criterion has also been chosen for the characteristics of the object, without taking disturbances into account, and is in principle realizable.

Fig. 1

Let us consider three cases.

1. Disturbances changing the characteristics of the object can be measured. The basic scheme is shown in Fig. 1 (without the dashed line). Here $W_2(P)$ is the transfer function of the object; $KW_1(P)$ and $W_3(P)$ are the transfer functions of the control system and the stabilizing device. $KW_1(P)$ and $W_3(P)$ are chosen so that the optimal operating regime in the absence of disturbances F is achieved for sufficiently large K. The structure is chosen so that stability is achieved with unlimited increase of K.

We shall prove the following proposition. The structure of Fig. 1 without taking disturbances into account, for a sufficiently large amplification coefficient K, is equivalent to the structure of Fig. 1 when the disturbances, additionally applied to the input of the stabilizing device (shown by the dashed line), are taken into

Fig. 2

Figure 2: Fig. 2

account, for sufficiently large K.

Indeed, according to Fig. 1, when disturbances are neglected,

$$x_{\text{out}} = \lim_{K\to\infty} \frac{\dfrac{KW_1(P)}{1+KW_1(P)W_3(P)}\,W_2(P)}{1+\dfrac{KW_1(P)W_2(P)}{1+KW_1(P)W_3(P)}}\,x_{\text{in}} = \frac{W_2(P)}{W_3(P)+W_2(P)}\,x_{\text{in}}. \quad (1)$$

* It is meant that the time during which the parameters change is considerably greater than the time of the given transient process.

Taking the disturbances into account and under the condition that these disturbances, in addition to the object, are also applied to the input of the stabilizing element, we have

$$[1+KW_1(P)W_3(P)]x_{\text{out}} = KW_1(P)W_2(P)x_{\text{in}} - KW_1(P)W_2(P)x_{\text{out}} -$$
$$-KW_1(P)W_2(P)W_3(P)F + W_2(P)F + KW_1(P)W_2(P)W_3(P)F \quad (2)$$

or

$$x_{\text{out}} = \lim_{K\to\infty} \frac{KW_1(P)W_2(P)+W_2(P)F}{1+KW_1(P)W_2(P)+KW_1(P)W_3(P)}x_{\text{in}} = \frac{W_2(P)}{W_3(P)+W_2(P)}x_{\text{in}}, \quad (3)$$

which was to be proved. It follows from what has been obtained that such a system will behave as a self-adjusting one in the sense that its characteristics will remain unchanged despite the presence of disturbances (interference). It is important to note that compensation of external interference is achieved here automatically, without the need to act on the system parameters, as is done, for example, in the series of works ([3]-[5]).

Fig. 2

2. Disturbances that change the characteristics of the object cannot be measured. The solution in this case is constructed as follows. Suppose that the characteristics of the object are known to us when the disturbance is absent. If the control system for this case is chosen in the same way as shown in Fig. 1, then in the absence of disturbances and with sufficiently large K we shall have

$$x_{\text{out}} = \frac{W_2(P)}{W_2(P) + W_3(P)} x_{\text{in}}.$$

We supplement the scheme of Fig. 1 with the following elements (see Fig. 2). In parallel with the real object, whose characteristics change under the action of external disturbances F, we include an object model with transfer function $W_2(P)$. The difference $x_{\text{out}} - x'_{\text{out}}$, through a converting device consisting of amplifiers with large gain coefficients, enclosed by passive networks so that the resulting transfer function will be $W_{\text{pr}}(P) = 1/W_2(P)$, is applied to the input of the stabilizing device. Writing the transfer function and passing to the limit as $K \to \infty$, we have

$$\begin{gathered}[1 + KW_1(P)W_3(P) + KW_1(P)W_2(P)]x_{\text{out}} = \\ = KW_1(P)W_2(P)x_{\text{in}} - KW_1(P)W_2^2(P)W_{\text{pr}}(P)W_3(P)F + W_2(P)F + \\ + KW_1(P)W_2(P)W_3(P)F,\end{gathered} \tag{4}$$

$$x_{\text{out}} = \lim_{K \to \infty} \frac{KW_1(P)W_2(P)x_{\text{in}} + W_2(P)F}{1 + KW_1(P)W_3(P) + KW_1(P)W_2(P)} = \frac{W_2(P)}{W_2(P) + W_3(P)} x_{\text{in}}. \tag{5}$$

Elimination of the influence of interference in this case could be achieved by the methods set forth in ([6],[7]). It is not difficult, however, to show that in the present case the system will be more noise-resistant to disturbances entering the input together with the useful signal.

3. The change of the object parameters occurs on account of the internal properties of the object. This case applies to objects in which, during operation, the parameters of the object itself may vary within wide limits. The problem here is to choose such a structure of the control system for which the prescribed regime would depend little on changes in the parameters of the object. As an estimate of the degree of influence of changes in the object parameters on the regime may serve

the sensitivity quantity $S_{W_2(P)}^{K(P)}$:

$$S_{W_2(P)}^{K(P)} = \frac{dK(P)/K(P)}{dW_2(P)/W_2(P)} = \frac{dK(P)}{dW_2(P)} \frac{W_2(P)}{K(P)}, \tag{6}$$

where $K(P)$ is the transfer function of the entire system, and $W_2(P)$ is the transfer function of the plant.* The smaller the quantity $S_{W_2(P)}^{K(P)}$, the less sensitive the dynamic properties of the system as a whole are to changes in the properties of the plant. In this sense it is natural to consider the system ideal

Fig. 3

Figure 3: Fig. 3

if the quantity $S_{W_2(P)}^{K(P)}$ does not depend on the characteristics of the plant, or if $S_{W_2(P)}^{K(P)} \to 0$.

Let us prove the following proposition. Structures that are stable under an unlimited increase of the gain coefficient, for which stability is achieved by introducing derivatives in the manner considered in (7), belong to the class of self-adjusting systems in the above sense. A typical case of structures of this kind may be represented by Fig. 3 (8).

Fig. 3

In this case

$$K(P) = \frac{x_{\text{out}}}{x_{\text{in}}} = \frac{\dfrac{K}{W_3(P)} W_2(P)}{1 + \dfrac{K}{W_3(P)} W_2(P)}; \tag{7}$$

$$S_{W_2(P)}^{K(P)} = \frac{dK(P)}{dW_2(P)} \frac{W_2(P)}{K(P)} = \frac{1}{1 + \dfrac{K}{K_3(P)} W_2(P)}; \tag{8}$$

$$\lim_{K \to \infty} S_{W_2(P)}^{K(P)} = 0, \tag{9}$$

i.e., the selected mode does not depend on changes in the properties of the plant.

For the general case of structures stable at an unlimited gain coefficient, the independence of the system's operating mode from changes in the characteristics of the plants is achieved by the additional introduction of a model of the plant with invariant characteristics. If the transfer function of the model is denoted by $W_2'(P)$, then

$$\lim_{K \to \infty} x_{\text{out}} = \frac{W_2'(P)}{W_3(P) + W_2'(P)} x_{\text{in}} \tag{10}$$

and does not depend on $W_2(P)$.

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* Naturally, sensitivity may be considered with respect to some parameter, and not to the entire transfer function of the plant.

Note: Figure translations are in progress. See original paper for figures.