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**Abstract**

**Full Text**

**Physics**

**S. V. Tyablikov and E. N. Yakovlev**

## **On a Certain Generalization of the Spin-Wave Method**

*(Presented by Academician N. N. Bogolyubov on 25 XII 1961)*

The range of applicability of the usual spin-wave approximation <sup>(1)</sup>, often used in the theory of magnetism, is limited to the region of low temperatures. The results of Dyson <sup>(2,3)</sup>, as well as those of Oguchi <sup>(4)</sup> and Oguchi <sup>(5)</sup>, made it possible to extend the range of applicability of this approximation to temperatures of the order of  $\frac{1}{4}$  of the Curie temperature. However, in the works mentioned the problem considered was mainly that of calculating the statistical sum. Below we shall present a simpler method for calculating the energy of spin waves and their damping, based on the perturbation-theory solution of a chain of equations for two-time temperature Green functions <sup>(6)</sup>.

We shall restrict ourselves to the case of an isotropic ferromagnet and to the inclusion of interactions only between nearest neighbors. Then the Hamiltonian of the spin system may be written in the following form:

$$\mathcal{H} = -\mu H \sum_{(f)} S_f - I \sum_{(f,\delta)} S_f^\alpha S_{f+\delta}^\alpha, \quad (1)$$

where  $f$  is the vector of a lattice site;  $\delta$  is the vector connecting this site with its nearest neighbor;  $I$  is the exchange integral;  $\mu$  is the magnetic moment of a site;  $S_f^\alpha$  is a component of the spin operator belonging to the site  $f$ .

Following the work of Oguchi <sup>(5)</sup>, we shall consider the value of  $S$  sufficiently large ( $S \gg 1$ ) and, accordingly, use expansions in powers of  $S^{-1}$ . Using further the Holstein-Primakoff transformation <sup>(7)</sup>, we give the Hamiltonian (1) the form <sup>(5)</sup>

$$\begin{aligned}
\mathcal{H} = & A + \sum_{(k)} E_k a_k^+ a_k + \\
& + \varepsilon \frac{Iz}{2N} \sum_{(k_1, k_2, k_3, k_4)} \Delta(k_1 + k_2 - k_3 - k_4) \left[ (\gamma_{k_1} + \gamma_{k_2} - 2\gamma_{k_4 - k_2}) + \right. \\
& + \left. \frac{\varepsilon}{8S} (\gamma_{k_1} + \gamma_{k_4}) \right] a_{k_1}^+ a_{k_2}^+ a_{k_3} a_{k_4} + \varepsilon^2 \frac{Iz}{16SN^2} \sum_{(k_1, \dots, k_6)} \Delta(k_1 + k_2 + k_3 - k_4 - k_5 - k_6) \times \\
& \times (\gamma_{k_1} + \gamma_{k_6} - 2\gamma_{k_3 - k_5 - k_6}) a_{k_1}^+ a_{k_2}^+ a_{k_3}^+ a_{k_4} a_{k_5} a_{k_6},
\end{aligned} \tag{2}$$

where the following notation has been introduced:

$$\begin{aligned}
A = -\mu HN + 2IzS^2 N, \quad E_k = \mu H + 2IzS(1 - \gamma_k), \\
\gamma_k = \frac{1}{z} \sum_{(\delta)} e^{i(k, \delta)}, \quad \Delta(x) = \begin{cases} 1, & x = 0, \\ 0, & x \neq 0, \end{cases}
\end{aligned} \tag{3}$$

and where  $N$  is the number of sites in the lattice;  $z$  is the number of nearest neighbors;  $a_k^+$ ,  $a_k$  are Bose creation and annihilation operators for a spin wave with wave vector  $k$  and energy  $E_k$ ;  $\varepsilon$  is a formal small parameter introduced for convenience (in the final results one should put  $\varepsilon = 1$ ). We note that in the Hamiltonian (2) the kinematic interaction of spin waves (see (2,3)) is taken into account up to and including terms of order  $S^{-2}$ .

Following work [6], we now construct a chain of equations for the Fourier transforms of the Green functions with respect to time. For the first function  $G_1$  the equation has the form

$$\begin{aligned}
(E - E_k)G_1(k) = & \frac{i}{2\pi} + \\
& + \varepsilon \frac{Iz}{2N} \sum_{(k_2, k_3, k_4)} \Delta(k + k_2 - k_3 - k_4) \left[ (\gamma_k + \gamma_{k_2} + 2\gamma_{k_4} - 2\gamma_{k_2 - k_4} - 2\gamma_{k - k_4}) \right. \\
& + \left. \frac{\varepsilon}{8S} (\gamma_k + \gamma_{k_2} + 2\gamma_{k_4}) \right] \langle\langle a_{k_2}^+ a_{k_3} a_{k_4} | a_k^+ \rangle\rangle + \\
& + \varepsilon^2 \frac{Iz}{16SN^2} \sum_{(k_2, \dots, k_6)} \Delta(k + k_2 + k_3 - k_4 - k_5 - k_6) \times \\
& \times (\gamma_k + \gamma_{k_2} + \gamma_{k_3} + 3\gamma_{k_6} - 4\gamma_{k_3 - k_5 - k_6} - 2\gamma_{k - k_5 - k_6}) \langle\langle a_{k_2}^+ a_{k_3}^+ a_{k_4} a_{k_5} a_{k_6} | a_k^+ \rangle\rangle \\
(G_1(k) = & \langle\langle a_k | a_k^+ \rangle\rangle).
\end{aligned} \tag{4}$$

As we see, the right-hand side contains the second Green function (terms of order  $\varepsilon$  and  $\varepsilon^2$ ) and the third Green function (the last term of order  $\varepsilon^2$ ). Therefore, in order to determine from equation (4), with accuracy up to and including quantities of order  $\varepsilon^2$ , the function  $G_1$ , or the mass operator  $M_1$  for it, it is necessary to find from the corresponding equations the second Green function with accuracy up to quantities of order  $\varepsilon$ , and the third function in zeroth order in  $\varepsilon$ .

Here we shall be interested in the first nontrivial temperature corrections; therefore, in the equations for the Green functions further simplifications can be made. Namely, in equation (4) the last term may be omitted, since even upon substituting the zeroth approximation for the third Green function it gives a contribution to the mass operator of order  $\vartheta^{3/2}$  ( $\vartheta$  is the temperature in energy units). In the equation for the second Green function, by analogous considerations, the terms containing the third and fourth Green functions are omitted. As a result, with the adopted degree of accuracy, the equation for the second function is written in the form

$$\begin{aligned}
 & (E + E_{k_2} - E_{k_3} - E_{k_4}) \langle\langle a_{k_2}^+ a_{k_3} a_{k_4} | a_k^+ \rangle\rangle = \\
 & = \frac{i}{2\pi} \bar{n}_{k_2} \{ \Delta(k_2 - k_3) \Delta(k - k_4) + \Delta(k_2 - k_4) \Delta(k - k_3) \} + \\
 & + \varepsilon \frac{Iz}{2N} \sum_{(k'_2, k'_3, k'_4)} \Delta(k_4 + k'_2 - k'_3 - k'_4) \Delta(k'_2 - k_3) \times \\
 & \times (\gamma_{k_4} + \gamma_{k'_2} + 2\gamma_{k'_4} - 2\gamma_{k'_2 - k'_4} - 2\gamma_{k_3 - k'_4}) \langle\langle a_{k_2}^+ a_{k'_3} a_{k'_4} | a_k^+ \rangle\rangle,
 \end{aligned} \tag{5}$$

where the notation has been introduced

$$\bar{n}_k = \left[ \exp\left(\frac{E_k}{\vartheta}\right) - 1 \right]^{-1}. \tag{6}$$

We define the mass operator  $M_1$  for the first Green function  $G_1$  by the equation:

$$(E - E_k - M_{1k}(E)) G_1(k) = \frac{i}{2\pi}. \tag{7}$$

Using equations (4), (5), (7) and the definition of  $E_k$  (3), it is not difficult to obtain, for  $E = E_k$ , the expression  $M_k$

$$M_{1k}(E_k) = M'_k - iM''_k, \tag{8}$$

where

$$M'_k = \varepsilon \frac{2Iz}{N} \sum_{(k_2)} (\gamma_k + \gamma_{k_2} - \gamma_0 - \gamma_{k-k_2}) \bar{n}_{k_2} +$$

$$\begin{aligned}
 & +\varepsilon^2 \frac{Iz}{4SN} \sum_{(k_2)} (\gamma_k + \gamma_{k_2}) \bar{n}_{k_2} - \\
 & -\varepsilon^2 \frac{Iz}{4SN^2} \sum_{(k_2, k_3)} P \frac{(\gamma_k + \gamma_{k_2} + \gamma_{k_3} + \gamma_{k+k_2-k_3} - 2\gamma_{k-k_3} - 2\gamma_{k_2-k_3})^2}{\gamma_k + \gamma_{k_2} - \gamma_{k_3} - \gamma_{k+k_2-k_3}} \bar{n}_{k_2} \quad (9)
 \end{aligned}$$

( $P$  is the principal-value symbol),

$$M_k'' = \varepsilon^2 \pi \frac{Jz}{SN^2} \sum_{(k_2, k_3)} \bar{n}_{k_2} (\gamma_{k_3} + \gamma_{k+k_2-k_3} - \gamma_{k-k_3} - \gamma_{k_2-k_3})^2 \delta(\gamma_k + \gamma_{k_2} - \gamma_{k_3} - \gamma_{k+k_2-k_3}). \quad (10)$$

The spectrum of “one-particle” elementary excitations of the system (spin waves) is determined by the poles of the function  $G_1$

$$\varepsilon_k = E_k + M_k' = \mu H + 2IzS(1 - \gamma_k) + M_k'. \quad (11)$$

Noting that  $N^{-1} \sum_{(k)} \gamma_k = 0$ , it is not difficult to see that there is no gap in the spectrum of spin waves, apart from the trivial one ( $E_0 = \mu H$ ,  $M_0' = 0$ ), and that, as should be the case, there is no damping of spin waves with  $k = 0$  ( $M_0'' = 0$ ).

In the region of low temperatures, small wave vectors play an essential role. Therefore, for  $M_k'$  one can obtain an approximate expression by expanding the corresponding  $\gamma_k$  in formula (8) in powers of  $k$ . As a result we obtain an approximate expression which it is convenient to write in the following form:

$$M_k' \cong -2IzS(1 - \gamma_k)\alpha\tau^{5/2}, \quad \tau = \frac{\vartheta}{8\pi IzSa_2}, \quad \alpha = \frac{\varepsilon\pi}{S} \left(1 + \frac{\varepsilon a_1}{S}\right) \zeta_{5/2}, \quad (12)$$

where  $\zeta_p$  is the Riemann zeta function;  $a_1, a_2$  are numbers of order unity, depending on the geometry of the lattice. For a simple cubic lattice  $a_1 \cong 0.2$ ,  $a_2 = 1/6$ ,  $z = 6$ . From (10), (11) we see that the energy of spin waves

$$\varepsilon_k = 2ISz(1 - \gamma_k)(1 - \alpha\tau^{5/2})$$

decreases with increasing temperature as  $\tau^{5/2}$ . In this respect there is a difference from paper (8) and other similar papers, where, owing to the proportionality of  $\varepsilon_k$  to the relative magnetization,  $\varepsilon_k$  decreases as  $\tau^{3/2}$ . This is connected with the comparatively low accuracy of the first approximation used in (8).

Using formulas (11), (12) for the relative magnetization, it is easy to obtain expansions in temperature which coincide with the corresponding expansions of papers <sup>(2, 3, 5)</sup> for  $S \gg 1$ .

In the same way we obtain an approximate expression for the imaginary part of the mass operator. For a simple cubic lattice and spin waves with energy greater than the average,

$$M'' \cong \varepsilon^2 \frac{I\zeta_{5/2}}{2S\pi} \tau^{5/2} (ka)^3, \quad (13)$$

and for spin waves with energy less than the average,

$$M''_k \cong \varepsilon^2 \frac{8I\zeta_3}{3S} \tau^{6/2} (ka)^2, \quad (14)$$

where  $a$  is the lattice constant. The average damping of spin waves will be of order  $10^2 IS^{-1} \tau^4$ . We note that earlier in <sup>(2)</sup> the estimate  $M''_k \sim \tau^{5/2}$  was obtained.

From (13), (14) we see that the damping of spin waves tends to zero as  $k \rightarrow 0$  and that at low temperatures it is small for all  $k$ . The region of applica-

...the applicability of the spin-wave approximation is determined by the condition that the damping of the spin wave be small in comparison with its energy, i.e., by the inequality  $M''_k \ll \varepsilon_k$ . Taking into account that  $\tau \sim \vartheta/\vartheta_C$ , where  $\vartheta_C$  is the Curie temperature, we see that this requirement is fulfilled for  $\tau \ll 1$ ; in practice it is satisfied up to temperatures of the order of 1/4 of the Curie temperature.

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*Note: Figure translations are in progress. See original paper for figures.*

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