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Abstract

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ON THE ASYMMETRY OF SPECTRAL LINES OF HYDROGEN IN A PLASMA

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One of the methods for measuring the density of charged particles in a dense plasma is based on comparing the experimental profile of spectral lines with a calculated one. In this connection, recently a number of experiments have been carried out to study the contours of the spectral lines of hydrogen in the plasma of an arc burning in an atmosphere of helium and argon ([1]), and also in the plasma of a shock tube ([2]). The experiments of N. N. Sobolev and co-workers convincingly show that, at a noticeable concentration of charged particles in the plasma, $N \gtrsim 10^{16}$ cm^{-3}, an asymmetry is observed in the contour of the lines H_β and H_δ, expressed in the greater intensity of the "blue" maximum of the profile as compared with the "red" one, and also in an asymmetry of the positions of the intensity maxima relative to the center of the line. Thus, at $N \sim 10^{17}$ cm^{-3}, the change in intensity at the maxima amounts to $\sim 10\%$ for the line H_β.

It is known that a homogeneous electric field, which causes the Stark effect, cannot lead to the indicated asymmetry of the lines. The presence of an asymmetry effect at sufficiently high densities points to the necessity of taking into account the inhomogeneity of the field.

It should be noted that existing theories of the shape of spectral lines, in particular the recent work of Griem, Kolb, and Shen ([3]), cannot explain the observed asymmetry. Both of the indicated characteristics, as well as the distance between the maxima, are functions of the density of the charged component of the plasma, which, as will be shown, leads to a new method for measuring the density of charged particles in a plasma.

At the indicated densities one may use the nearest-neighbor approximation. Let a hydrogen ion be located at a distance R from the radiating atom. Its action on the atom may be treated quasi-statically. Indeed, for this it is necessary that the lifetime of the atom on the given Stark sublevel be small in comparison with the time of appreciable displacement of the ion. It is easy to see that this condition is written in the form

$$N^{2/3} \gg \sqrt{\frac{m}{M}}\,\frac{1}{\pi n^4 \lambda_e^2}, \tag{1}$$

where m/M is the ratio of the electron and proton masses, n is the principal quantum number, and λ_e is the de Broglie wavelength of the electrons causing impact perturbation. For $T \approx 1$ eV, $N \gtrsim 10^{15}$ cm^{-3}, condition (1) is fulfilled with a large margin. The energy of interaction of the ion with the atom, with allowance for the quadrupole term, in the usual notation has the form ($R \gg n^2 a_0$)

$$U(R) = -\frac{e^2 r \cos\theta}{R^2} - \frac{e^2 r^2}{2R^3}\left(3\cos^2\theta - 1\right). \tag{2}$$

The first term in (2), for fixed R, gives a symmetric splitting of the spectral lines owing to the linear Stark effect. Writing the quadrupole interaction in parabolic coordinates,

$$\Delta U = -\frac{e^2}{4R^3}\left(\xi^2 + \eta^2 - 4\xi\eta\right), \tag{3}$$

we obtain the shift of the Stark components according to perturbation theory, roughly speaking, proportional to n^4:

$$\lambda E_{\mathrm{kv}} = -\frac{e^2 a_0^2}{2R^3}\, n^2\left[6(n_1 - n_2)^2 - n^2 + 1\right], \tag{4}$$

where n_1, n_2 are the "electric" quantum numbers in quantization in parabolic coordinates.

From formula (4) one can already obtain a qualitative idea of the enhancement of the intensity in the short-wavelength part of the line, since the shift of the Stark components leads to their denser "clustering" in the violet part of the spectrum.

The intensities of the Stark components in the dipole approximation were calculated by Schrödinger [4]. We shall obtain the intensities taking into account the quadrupole interaction. The total intensity of the dipole period $n' \to n$ has the form:

$$I_{n'n} = \frac{64\pi^4 e^2 \nu^4}{3c^3}\left|r^{\,n_1 n_2 m}_{\,n_1' n_2' m'}\right|^2, \tag{5}$$

where ν is the transition frequency; n' and n are the sets of quantum numbers of the initial and final states, and $r^{\,n_1 n_2 m}_{\,n_1' n_2' m'}$ is the coordinate matrix element

$$r_{n_1' n_2' m'}^{n_1 n_2 m} = \left\langle \psi^*_{n_1' n_2' m'} \, r \, \psi_{n_1 n_2 m} \right\rangle , \tag{6}$$

which is nonzero for $\Delta m = 0$ for radiation polarized parallel to the field (π-transitions), and $\Delta m = \pm 1$ for polarization perpendicular to the field (σ-transitions).

Since the perturbation ΔU does not depend on the angle φ, and hence also not on m, and also exceeds the fine-structure splitting, there is no need for the secular equation of perturbation theory. In first-order Schrödinger perturbation theory

$$\psi_{n_1 n_2 m} = \psi^{(0)}_{n_1 n_2 m} + \psi^{(1)}_{n_1 n_2 m}, \tag{7}$$

where $\psi^{(0)}_{n_1 n_2 m}$ are the wave functions of the Stark components in parabolic coordinates, unperturbed by the quadrupole interaction, and $\psi^{(1)}_{n_1 n_2 m}$ has the form

$$\psi^{(1)}_{n_1 n_2 m} = + \frac{n a_0}{2R} \times$$

$$\times \left[\sqrt{(n_1+1)(n-n_1-1) n_2 (n-n_2)} \, \psi^{(0)}_{n_1+1,\, n_2-1,\, m} - \right.$$

$$\left. - \sqrt{n_1 (n-n_1)(n_2+1)(n-n_2-1)} \, \psi^{(0)}_{n_1-1,\, n_2+1,\, m} \right] . \tag{8}$$

Table 1

Initial state	Final state	p_k	q_k	α_k	β_k	γ_k
111	001	0	228	0	0	0
111	010	−2	252	144	1	−14
111	100	+2	252	144	1	+14
120	010	+2	156	9	1	+118
210	100	−2	156	9	1	−118
210	001	−4	132	48	19	−252
102	001					
120	001	+4	132	48	19	252
012	001					
120	100	+6	156	84	1	+22/3
210	010	−6	156	84	1	−22/3
201	100	−6	432	84	7	−46
021	010	+6	432	84	7	+46
021	001	+8	−156	384	1	0

Initial state	Final state	p_k	q_k	α_k	β_k	γ_k
201	001	−8	−156	384	1	0
201	010	+10	−432	12	1	+34
021	100	+10	−432	12	1	−34
300	100	−10	−612	361	1	+2
030	010	+10	−612	361	1	−2
030	001	+12	−636	16	1	−36
300	001	−12	−636	16	1	36
300	010	−14	−612	1	1	70
030	100	+14	−612	1	1	−70

In doing so, only transitions without a change in the principal quantum number are taken into account, i.e., with $n = n'$, since the contribution of transitions with $n \neq n'$ is very small.

For the densities considered, it may be assumed that the population of the Stark levels is proportional to their statistical weights. This assumption is confirmed by experiment ([1]). The results of the calculation of ΔE_k and of the intensities of the individual components are given in Table 1. Here

$$\Delta E_k = \hbar \left[\omega_k - \omega_{42}^0\right] = \frac{3}{2}\frac{e^2 a_0^2}{R^2} P_k + \frac{e^2 a_0^2}{2R^3} q_k,$$

$$I_k = \alpha_k \left(\beta_k + \frac{\gamma_k}{R_k}\right)$$

is given in relative units.

It is not difficult to see that, in the nearest-neighbor approximation, the probability of the position of the Stark component k, in atomic units, is

$$W_k(\omega)\, d\omega = \frac{R_k}{2R_0^3} \frac{\exp\left[-\left(\frac{R_k}{R_0}\right)^3\right]}{\left|D_k + \dfrac{6C_k}{R_k}\right|}\, d\omega, \tag{9}$$

where, for the H_β line,

$$C_k = 4(n_1 - n_2)^2 - (n_1' - n_2')^2 - \frac{19}{2}, \qquad D_k = 2(n_1 - n_2) - (n_1' - n_2');$$

$$R_0 = \left({}^{3}/_{4\pi N}\right)^{1/3}$$

is the mean distance between particles:

$$R_k(\omega) \simeq R_k^{(0)} + R_k^{(1)} - \frac{3}{2}\frac{R_k^{(1)\,2}}{R_k^{(0)}}, \qquad R_k^{(0)} = \sqrt{\frac{3D_k}{\omega_{42}^0 - \omega}}, \qquad R_k^{(1)} = \frac{2C_k}{D_k}.$$

Then the total intensity as a function of ω (the line contour) has the form

$$I(\omega) = \sum_k I_k\left[R_k(\omega)\right] W_k(\omega). \tag{10}$$

The positions of the intensity maxima are determined from the condition $dI/d\omega = 0$.

Table 2

N, cm^{-3}	ΔI_{theor}, %	ΔI_{expt}, %	$\lambda^{\text{c}}_{\text{max}}$, Å	$\lambda^{\text{kr}}_{\text{max}}$, Å	$\Delta\lambda^{\text{kr}}$, Å	$\Delta\lambda^{\text{c}}$, Å	$\frac{\lvert\Delta\lambda^{\text{c}}\rvert - \lvert\Delta\lambda^{\text{kr}}\rvert}{\lvert\Delta\lambda^{\text{c}}\rvert}$, %
10^{16}	4.6	~ 4	4859.2	4861.1	0.92	−0.96	4.2
$5 \cdot 10^{16}$	7.6	~ 6.5	4857.3	4862.8	2.61	−2.81	7.0
10^{17}	9.8	~ 10	4855.7	4864.3	4.15	−4.47	7.2
10^{18}	23.3	–	4838.7	4879.2	19.08	−21.47	11.1

In Table 2, for the H_β line, values are given for the relative change of the intensities at the maxima,

$$\Delta I = \frac{I_{\text{c}} - I_{\text{kr}}}{I_{\text{c}}},$$

and the corresponding experimental values from work ([2]), as well as the shift of the maxima relative to the center of the line ($\Delta\lambda$). Data for the H_δ line are not given because, although the effect in this line exceeds the effect for H_β, it proves to be less convenient, since its intensity is smaller than the intensity of the H_β line and its recording requires longer exposures.

The dependence of ΔI on density in the interval $N \sim 10^{16} - 10^{17}$ cm^{-3} proves to be a convenient monotonic function for calculations. It should be noted that the relative change of the intensity at the maxima depends extremely weakly on the method of averaging; therefore, the indicated method of measuring the density

the plasma may turn out to be more accurate than the method of measuring the density from the line width, taking into account the inaccuracy of measuring the latter and the imperfection of existing theories for the line width at given densities.

The dependence of the distance between the maxima in the function on the density is in qualitative agreement with experiment; however, the theory set forth here cannot claim to give a good value for the width, owing to the crudeness of the distribution function used for the ion field (although it is better than the Holtsmark one for the densities in question) and to the absence of allowance for impact broadening by electrons.

In Griem' s paper ([5]) the asymmetry of hydrogen lines was explained by taking account of the quadratic Stark effect. It is evident that the quadratic Stark effect gives a contribution of the next order in a_0/R in comparison with the quadrupole term and is not the principal effect.

In the theory presented here nothing can be said about the center of the line, but this is of no concern to us, since the intensity maxima lie sufficiently far from its center, so that the ratio of the shift of the maximum to the impact width at $T \sim 1$ eV and $N \sim 10^{17}$ cm^{-3}

$$\frac{\Delta_m}{\hbar\gamma} \simeq \frac{6}{n^2}\frac{R_0}{\lambda_e} \simeq 30 \gg 1.$$

The contribution of the Doppler effect,

$$\gamma_D = \frac{\omega_{42}}{c}\sqrt{\frac{2kT}{m_H}} \sim 0.2 \text{ Å},$$

is negligibly small at this temperature. Further improvement of the theory should concern the distribution function $W(\omega)$ and allowance for electron broadening.

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