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Abstract

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BASIC PROPERTIES OF THE SUPERFLUID MODEL OF THE NUCLEUS

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In ([1]) a superfluid model of the nucleus was proposed, and in ([2,3]) calculations were carried out of the single-particle levels of a number of strongly deformed elements both in the rare-earth and in the transuranium regions. In ([4]) the influence of pairing correlations on the probabilities of β transitions in strongly deformed nuclei was analyzed. In these papers it was shown that the results obtained on the basis of the superfluid model of the nucleus correctly reflect many regularities in the behavior of complex nuclei.

In the present note we investigate the general properties of the superfluid model of the nucleus, namely: the dependence of the characteristics of superfluid states and of the behavior of the levels of an even system on the magnitude of the pairing-interaction constant G, the superfluid properties of two-quasiparticle excited states of the system, the specific features of 0+ states, etc.

We study the behavior of the ground state and of two-quasiparticle excited states as the pairing-interaction constant G is increased, using as an example a neutron system with $N = 106$, as in Hf^{178}. From the experimental value of the pairing energy and for such a position of the mean-field levels that the single-particle level energies of neighboring odd nuclei calculated on the basis of the superfluid model of the nucleus agree with the experimental values, we find $G = 0.020\,\hbar\omega_0^0$ ($\hbar\omega_0^0 = 41A^{-1/3}$ MeV).

Let us consider the behavior of the ground state of the system as G increases. For $G = 0.012\,\hbar\omega_0^0$ the correlation function C is very small, and the energy of the ground state is approximately equal to its value at $G = 0$. Thus, for $G = 0.012\,\hbar\omega_0^0$ pairing correlations are practically absent. For $G = 0.016\,\hbar\omega_0^0$, $C = 0.058\,\hbar\omega_0^0 = 0.42$ MeV, and the energy decreases by $0.017\,\hbar\omega_0^0 = 0.12$ MeV in comparison with the case $G = 0$, although the negative potential energy C^2/G changes more strongly, namely $C^2/G = 1.5$ MeV.

In Fig. 1 we present the values of the ground-state energy and of the gap $2C$ for G equal to 0.016; 0.020; $0.024\,\hbar\omega_0^0$. For $G = 0.028\,\hbar\omega_0^0$ the ground-state energy decreases by $0.70\,\hbar\omega_0^0 = 5.1$ MeV, and $C = 0.28\,\hbar\omega_0^0 = 2.1$ MeV. For $G = 0.032\,\hbar\omega_0^0$ the correlation function of the ground state is $C = 0.37\,\hbar\omega_0^0 = 2.7$ MeV, and the energy decreases by $1.17\,\hbar\omega_0^0 = 8.5$ MeV, with $C^2/G = 30.5$ MeV, i.e., the increase in the energy of the system due to the smearing of the Fermi-surface boundary is significant and amounts to 22 MeV.

Let us consider the behavior of the two-quasiparticle excited states of the system as a function of G. For this purpose, in Fig. 1, *b* we present the energies of the excited states calculated on an electronic computer on the basis of the superfluid model of the nucleus. In Fig. 1, *a* the energies of the excited states are given as calculated by the formula $\sqrt{C^2 + \{E(i) - \lambda\}^2} + \sqrt{C^2 + \{E(e) - \lambda\}^2}$, according to the original formulation of pairing correlations ([5,6]). Here

λ is the chemical potential, $E(i)$ is the energy of level i of the mean field. By $(k, k+2)$ we shall denote the state of the system with one quasiparticle on level k, and the second on level $k+2$, where k is the last occupied single-particle level of the mean field for $G = 0$. Let us note that in the superfluid model of the nucleus the number of particles is conserved on the average, and all calculated excited states refer to one and the same system of particles. In the case of [5,6], in the spec-

(Figure: Fig. 1)

Fig. 1

trum of excitations of the system, whose ground state consists of N particles, there are levels corresponding to systems of $N-2$, N, and $N+2$ particles. Thus, the states $(k, k-1)$ and $(k-2, k)$ shown in Fig. 1 refer to a system of $N-2$ particles; the states $(k, k+1)$, $(k, k+2)$, $(k-1, k+1)$, (k, k) refer to a system of N particles and, finally, the states $(k+1, k+2)$, $(k+1, k+4)$, $(k+1, k+1)$, and $(k+2, k+2)$ refer to a system consisting of $N+2$ particles.

The behavior of the energies of two-quasiparticle excited states as a function of G, calculated on the basis of the superfluid model of the nucleus, as is seen from Fig. 1, differs strongly at small G from their behavior in case *a*, where an increase in the energy of a number of the lowest states is observed with increasing G up to values $G = 0.020\,\hbar\omega_0$, associated with the incorrectness of the treatment. In case *b*, the energies of both the ground and the excited states decrease monotonically with increasing G, the degree of decrease of their energies being different. This difference is connected with one of the main features of the superfluid model of the nucleus, according to which changes in the superfluid properties of excited states are taken into account both relative to one another and in comparison with the ground state of the system.

Let us analyze the behavior of the correlation functions $C(k_1, k_2)$ of two-particle excited states; for this purpose, in Table 1 the changes are recorded

ratios $C(k_1, k_2)/C$ with increasing G. In the interval of values of G from 0.016 to $0.024\,\hbar\omega_0^0$, a large difference is observed in the values of the correlation functions of the excited states, while at $G = 0.028\,\hbar\omega_0^0$ and larger they differ only weakly from one another and are approximately 15–20% smaller than the value of the correlation function C of the ground state. Thus, in the superfluid model of the nucleus considered here, the difference between the correlation functions of excited states is very substantial at the value $G = 0.020\,\hbar\omega_0^0$, which corresponds to real residual nuclear forces.

One of the important results of calculations according to the superfluid model of the nucleus is the discovery that the energies of one, and in a number of cases several, excited states drop below the value of the gap $2C$. In [2] it was shown that in the excited state $(k, k+1)$ of an even system the superfluidity is considerably reduced. This is connected with the fact that, because of the Pauli principle, correlated pairs cannot occupy the levels k and $k+1$, and therefore in the states available for pairs, in strongly deformed nuclei, a considerable gap appears. Since below the gap the number of states is equal to the number of particles, and it is energetically unfavorable for pairs to occupy $k+2$ and higher levels, the superfluidity in the state $k, k+1$ is strongly reduced.

Table 1

G in units of $\hbar\omega_0^0$	0.016	0.020	0.024	0.028	0.032
$C(k, k+1)/C$	0	0.23	0.67	0.74	0.83
$C(k-1, k+1)/C$	0	0.46	0.70	0.78	0.84
$C(k, k+2)/C$	0	0.53	0.71	0.79	0.84
$C(k, k)/C$	0.56	0.62	0.71	0.79	0.84
$C(k-1, k)/C$	0.62	0.67	0.74	0.79	0.84
$C(k-2, k-1)/C$	0.72	0.75	0.78	0.81	0.85

Table 2

Energies of the states $(k, k+1)$

Nucleus	System	Gap $2C$ (MeV)	Spin and parity of the state $(k, k+1)$	Energy of the state $(k, k+1)$, calculation (MeV)	Energy of the state $(k, k+1)$, experiment (MeV)
W^{182}	Proton	1.61	2−	1.4	1.290
W^{182}	Neutron	1.89	4−	1.46	1.554
Hf^{178}	Proton	1.66	8−	1.03	1.148
Hf^{178}	Neutron	1.85	8−	1.46	1.480
Dy^{162}	Neutron	1.83	5−	1.32	1.485
Dy^{160}	Proton	1.9	2−	1.4	1.26
Gd^{156}	Proton	2.0	4+	1.45	1.511

As is seen from Table 2, the calculated values obtained for the energy of the state $(k, k+1)$ for a number of nuclei are noticeably smaller than the quantities $2C$ and are in good agreement with the corresponding experimental data. The agreement of theory with experiment with respect to the lowering of the energy of the state $(k, k+1)$ below the gap is one of the most important confirmations of the correctness of the initial assumptions of the superfluid model of the nucleus and testifies to its advantage in the investigation of the properties of strongly deformed nuclei in comparison with the original method of studying pair correlations.

Among the excited states of an even system, a special place is occupied by 0+ states with two quasiparticles on one and the same single-particle level of the mean field. The wave functions of these states are not orthogonal to one another and to the wave function of the ground state of the system. The methodological difficulties connected with conservation of the particle number on the average are, as it were, concentrated in the 0+ states, among which one state is superfluous. Let us analyze the dependence on G of the magnitudes of the nonorthogonality of the wave functions of the ground and excited 0+ states. As is seen from Table 3, as $G \to 0$ the states (k, k) and $(k+1, k+1)$ merge into one, while the others become mutually orthogonal; with increasing G, the nonorthogonality between the states (k, k) and $(k+1, k+1)$ decreases, and the nonorthogonality between the other states increases.

Let us roughly estimate the accuracy of the method; for this purpose we shall find the ratio $\Delta n/2\Omega$ of the mean-square fluctuation of the number of particles Δn to the number of states 2Ω of the mean field taken into account. We note that the functions characterizing the superfluid properties are most effective in the region of energy values (3—4)C above and below the level k. We find the ratio $\Delta n/2\Omega$ to be equal to 0.08 for an energy half-interval $3C$, 0.06 for $4C$, and 0.05 for $5C$; in the extremely small half-interval $2C$ the ratio $\Delta n/2\Omega = 0.12$. In solving the equations for finding C and λ, it is necessary to carry out the summation over an energy interval greater than (8—10)C, since otherwise the ratio $C(k)/C$ will be too small ($C(k)$ is the correlation function of the ground state of an odd system). Since for excited states Δn is always smaller than for the ground state, the errors of the method are of the order of 5% and in any case nowhere exceed 10%. Thus, the difference in the superfluid properties of the excited and ground states of an even system is substantial and lies outside the errors of the method.

Table 3

Nonorthogonality of 0+ states

	G in units of $\hbar\omega_0$					
	0.12	0.016	0.020	0.024	0.028	0.032
$\langle 0 \mid k-1, k-1 \rangle$	10^{-5}	0.13	0.18	0.12	0.07	0.03

	G in units of $\hbar\omega_0$					
$\langle 0 \mid k, k\rangle$	10^{-5}	0.27	0.24	0.11	0.05	0.02
$\langle k, k \mid k+1, k+1\rangle$	0.92	0.52	0.20	0.02	0.003	0.001
$\langle k, k \mid k+2, k+2\rangle$	0.004	0.07	0.07	0.02	0.005	0.002

Consequently, the specific features of the superfluid model of the nucleus are important at values $G = 0.020\,\hbar\omega_0$, corresponding to residual nuclear forces; at values of G twice as large these features would be absent, and the pair correlations would be so strong that they would at least mask the shell structure.

Let us note, finally, that the mathematical apparatus of the superfluid model is internally consistent and convenient for carrying out unambiguous quantitative calculations of a number of properties of specific nuclei.

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Note: Figure translations are in progress. See original paper for figures.