Physical Chemistry

1961

SovietRxiv

Abstract

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Physical Chemistry

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The Influence of a Magnetic Field on the Motion of Particles in Electrolyte Solutions

(Presented by Academician A. N. Frumkin on 6 XII 1960)

The question of the influence of a magnetic field on the motion of liquid or solid particles in electrolytes has so far been little studied. It is therefore of interest to consider some aspects of this motion. As will be shown below, the velocity of particles in an electrolyte under the action of a magnetic field is perpendicular to the electrophoretic velocity and proportional to the square of the particle radius. It is known that the electrophoretic velocity is proportional to the particle radius; therefore the results obtained in the present work may be used for a more accurate determination of particle sizes and for separating them by size. In addition, the results obtained may prove useful in studying the structure of particles—determining their viscosity, the magnitude of the surface charge, and the hardness of the surface layer. Problems of this kind may arise, for example, in biology.

Thus, let a particle, which we shall regard as a sphere of radius a, be placed in an electrolyte solution through which a current flows, produced by an electric field $\mathbf{E}$. Perpendicular to $\mathbf{E}$ a magnetic field $\mathbf{H}$ is applied. Far from the particle the fields $\mathbf{E}$ and $\mathbf{H}$ are uniform and constant.

We introduce a spherical coordinate system with origin at the center of the drop and polar axis z along $\mathbf{E}$. The azimuthal angle φ is measured from the zx plane; the y axis is directed along $\mathbf{H}$. In our frame of reference the particle is at rest, while the liquid at infinity moves with velocity U_0.

Let us first consider the case when no surface charge is formed on the particle. Assuming that no current flows through the particle, the distribution of the potential outside it is given by the formula:

$$\varphi = \left(r + \frac{a^3}{2r^2}\right) E \cos\theta. \tag{1}$$

Per unit volume of electrolyte in which a current of density

$$\mathbf{j} = -\chi \operatorname{grad} \varphi, \tag{2}$$

flows, there acts a volume force

$$\mathbf{F} = \frac{1}{c}[\mathbf{jH}].$$

Using (1) and (2), we obtain for the components of the force:

$$F_r = \frac{\chi EH}{c}\left(1 + \frac{a^3}{2r^3}\right)\sin\theta\cos\varphi; \qquad F_\theta = \frac{\chi EH}{c}\left(1 - \frac{a^3}{r^3}\right)\cos\theta\cos\varphi; \quad (3)$$

$$F_\varphi = -\frac{\chi EH}{c}\left(1 - \frac{a^3}{r^3} + \frac{3a^3}{2r^3}\sin^2\theta\right)\sin\varphi.$$

Since, for practically attainable fields, the velocity of motion is small, the motion is essentially viscous in character. In this case the hydrodynamic system of equations has the form:

outside the drop

$$\nabla p = \mu\Delta\mathbf{v} + \mathbf{F}, \qquad \operatorname{div}\mathbf{v} = 0; \quad (4)$$

inside the drop

$$\nabla p_1 = \mu_1\Delta\mathbf{v}_1, \qquad \operatorname{div}\mathbf{v}_1 = 0. \quad (5)$$

Boundary conditions at $r = a$:

$$v_r = v_{1r} = 0; \qquad v_\theta = v_{1\theta}; \qquad v_\varphi = v_{1\varphi}; \qquad p_{rr} = p_{1rr}; \qquad p_{r\theta} = p_{1r\theta}; \qquad p_{r\varphi} = p_{1r\varphi}. \quad (6)$$

In addition, the velocities of the internal and external liquids must be finite, respectively, at $r = 0$ and $r \to \infty$.

As is seen from (3), it is natural to seek the solution of (4) and (5) in the form

$$v_r = f(r)\sin\theta\cos\varphi; \quad v_\theta = g(r)\cos\theta\cos\varphi; \quad v_\varphi = \sin\varphi\,[h(r) + t(r)\sin^2\theta]; \quad (7)$$

$$p = \mu s(r)\sin\theta\cos\varphi.$$

To determine the functions of the radius, from (4) and (7) we obtain the system of equations

$$\begin{gathered} f'' + 4f'/r - s' = 4\lambda\,(1/a^3 + 1/2r^3); \\ g'' + 2g'/r - 2g/r^2 - 2t/r^2 + 2f/r^2 - s/r = 4\lambda\,(1/a^3 - 1/r^3); \\ f' - 2g/r + 2f/r + t/r = 0; \qquad h = -g; \\ t'' + 2t'/r - 6t/r^2 + 6\lambda/r^3 = 0; \end{gathered} \tag{8}$$

here

$$\lambda = \frac{\varkappa a^3}{4\mu c} EH.$$

The motion of the liquid inside the particle is described by analogous equations if in (8) one sets $\lambda = 0$.

Solving system (8) by ordinary methods, we obtain expressions for the velocities and pressure:

outside the drop

$$\begin{gathered} f = \frac{k}{r^3} + \frac{L}{r} + U_0; \qquad g = \frac{B-K}{2r^3} + \frac{L+\lambda}{2r} + U_0; \\ t = \frac{B}{r^3} + \frac{\lambda}{r}; \qquad s = \frac{L+\lambda}{r^2} - \frac{4\lambda r}{a^3}; \qquad h = -g; \end{gathered} \tag{9}$$

inside the drop

$$f_1 = M + Nr^2; \quad g_1 = M + r^2(2N + A/2); \quad t_1 = Ar^2; \quad s_1 = 10Nr; \quad h_1 = -g_1, \tag{10}$$

where $A, B, \ldots$ are constants of integration. In these expressions the requirement of finiteness of the velocity at $r = 0$ and $r \to \infty$ has been taken into account.

Determination of the constants of integration from the boundary conditions (6) leads to the system of equations

$$M + Na^2 = 0; \qquad K/a^3 + L/a + U_0 = 0; \qquad B/a^3 + \lambda/a = Aa^2;$$

$$\mu\,(K/a^5 + 2\lambda/a^2) = \mu_1 N;$$

$$\mu\left(\frac{3k-2B}{a^4} - \frac{\lambda}{a^2}\right) = \mu_1\left(3Na + \frac{aA}{2}\right); \tag{11}$$

$$\frac{B-K}{2a^3} + \frac{L+\lambda}{2a} + U_0 = M + \left(2N + \frac{A}{2}\right)a^2;$$

$$Aa\mu_1 = -\mu\left[\frac{4B}{a^4} - \frac{2\lambda}{a^2}\right].$$

The solution of this elementary, but cumbersome, system leads to the determination of the velocity of the particle relative to the electrolyte, equal in absolute value, in the chosen coordinate system, to the velocity of the electrolyte at infinity.

It turns out that the particle moves in a direction perpendicular to the electric and magnetic fields, with velocity

$$U_0 = \frac{\chi a^2 EH}{2\mu c}\left\{\frac{\mu + \mu_1}{2\mu + 3\mu_1}\right\}. \tag{12}$$

This motion (magnetophoresis) at $H = 10^4$ gauss and $j = 10^3$ has a velocity of the order of 0.1 cm/sec, i.e., the effect has a quite observable magnitude.

In work (1), an expression was obtained by other methods for the force acting on a solid particle under analogous conditions. For the case of an ideally polarizable particle this expression reduces to the formula

$$F = \frac{3V}{4c}\chi EH, \tag{13}$$

where V is the volume of the particle. Calculating from this, by Stokes' formula $F = 6\pi\mu a U$, the velocity of the particle, we obtain

$$U_0' = \chi a^2 EH/6\mu c, \tag{14}$$

which coincides exactly with our results if μ_1 is made infinite.

Let the particle now have a surface charge ε. In this case, in an electric field it will execute electrophoretic motion along E, the velocity of which, over broad ranges of values of ε, is at least an order of magnitude higher than the magnetophoretic velocity.

Assuming that the thickness of the double electric layer formed is much smaller than the particle radius, we have for the potential outside the drop the expression

$$\varphi = \left[r + \left(\frac{1}{2} - \frac{\varepsilon V_0}{\chi E a}\right)\frac{a^3}{r^2}\right] E\cos\theta. \tag{15}$$

Here V_0, as is clear from what was said above, may be taken equal to the electrophoretic velocity (2):

$$V_0 = \frac{\varepsilon E a}{2\mu + 3\mu_1 + \varepsilon^2/\chi}. \tag{16}$$

In addition to the volume current outside the drop, there is also a volume current inside the drop, caused by the motion of the charges of the inner plate of the double layer. The force acting in this case on a unit volume of the drop is equal to (3)

$$F_1 = \frac{1}{c\mu}[\mathbf{j}_1 \cdot \mathbf{H}]_x = \frac{2V_0 \varepsilon H}{ac}\,\mathbf{e}_x, \tag{17}$$

where $\mathbf{e}_x$ is the unit vector in the direction of the x-axis.

The force acting on the double layer from the magnetic field is equal to zero, since its plates have charges of opposite sign moving in one direction.

Thus, equations (4) remain unchanged, while in equations (5) the force (17) must be taken into account.

The insignificant boundary effects associated with the presence of a surface charge may be neglected, and conditions (6) are preserved.

In this case the solutions are:

outside the particle

$$\begin{gathered}
v_r = (K/r^3 + L/r + U)\sin\theta\cos\varphi; \\
v_\theta = \left(\frac{B-K}{2r^3} + \frac{L+q}{2r} + U\right)\cos\theta\cos\varphi; \\
v_\varphi = \left[-\left(\frac{B-K}{2r^3} + \frac{L+q}{2r} + U\right) + \left(\frac{B}{r^3} + \frac{q}{r}\right)\sin^2\theta\right]\sin\varphi; \\
p = \mu\left(\frac{L+q}{r^2} - \frac{4q}{R^3}r\right)\sin\theta\cos\varphi;
\end{gathered} \tag{18}$$

inside the particle [3]:

$$\begin{gathered}
v_{1r} = (M + Nr^2)\sin\theta\cos\varphi; \qquad v_{1\theta} = (M + Ar^2/a + 2Nr^2)\cos\theta\cos\varphi; \\
v_{1\varphi} = \left[-(M + Ar^2/2 + 2Nr) + Ar^2\sin^2\theta\right]\sin\varphi; \\
p_1 = \mu_1 \cdot 10Nr\sin\theta\cos\varphi + n\mu_1 r\sin\theta\cos\varphi; \\
q = 2\lambda\left(\frac{1}{2} - \frac{\varepsilon V_0}{\chi E a}\right); \qquad R = a\left[2\left(\frac{1}{2} - \frac{\varepsilon V_0}{\chi E a}\right)\right]^{1/3}; \qquad n = \frac{2V_0\varepsilon H}{a\mu_1}.
\end{gathered} \tag{19}$$

To determine the constants, from the boundary conditions we obtain a system of equations analogous to (11), whence we find the final expression for the velocity of magnetophoresis

$$U = U_0 \left[1 + \frac{8\mu + 15\mu_1}{3(\mu + \mu_1)} \frac{\varepsilon V_0}{\chi E a} \right], \tag{20}$$

where U_0 is determined from (12), and V_0 from (16).

It follows from (20) that the presence of a surface charge increases the velocity of magnetophoresis. An estimate shows that U may be 3-4 times greater than U_0, but it still remains possible in (15) to neglect U in comparison with V_0.

If the particle is solid, then there is no current inside it; consequently, there is no volume force (17). In this case the calculation gives, for the magnetophoresis velocity of a solid particle, the value

$$U = U_0(1 + \varepsilon V_\tau / \chi E a), \tag{21}$$

where V_τ is the electrophoresis velocity of a solid particle ([4]):

$$V_\tau = \frac{\varepsilon E d}{\mu + \varepsilon^2 d / a\chi}, \tag{22}$$

d is the thickness of the double layer. It has not yet been taken into account here that the force exerted by the magnetic field on the double layer is already nonzero, since one of its plates is immobile. It is not difficult to show that the correction to the velocity caused by this surface effect is an order of magnitude smaller than the value already obtained.

If the viscosity of the solution μ in (22) may be neglected in comparison with $\varepsilon^2 d / a\chi$, then from (21) we obtain

$$U_\tau = 2U_0. \tag{23}$$

It follows from (23) that in a magnetic field it is possible to separate uncharged solid particles from uncharged liquid ones.

In conclusion, the authors express their gratitude to Corresponding Member of the Academy of Sciences of the USSR V. G. Levich for valuable discussions.

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Received
2.XI.1960

CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.