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Abstract

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ON THE FREQUENCY OF FORMATION OF CAVITATION CAVITIES IN TURBULENT BOUNDARY LAYERS AND WAKE FLOWS

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The experimental investigations of K. K. Shal' nev (1), Knapp (2), and Daily (3) make it possible to conclude that the occurrence, growth, and collapse of cavitation cavities in turbulent boundary layers and wake flows is associated with pressure pulsations in the turbulent flow. Using this conclusion, one may picture the following scheme for the dynamics of cavitation cavities in a turbulent flow: at a certain instant a nucleus enters a region of rarefaction arising as a result of pressure pulsations, for which the value of the critical pressure P_k is greater than P. The bubble grows as long as the quantity $P_k - P$ remains positive. Having entered a region of increased pressure, the bubble collapses. Thus, the frequency of occurrence of cavitation cavities is determined by the frequency of appearance of rarefaction regions in which the pressure is below some critical pressure level, and also by the probability that a cavitation nucleus with this critical pressure level will enter the region of local pressure decrease.

Fig. 1. Distribution of pressures in the investigated volume of liquid. P_0 –hydrostatic pressure; P_k –critical cavitation pressure; $\bar{P} = P_0 - \alpha\rho u_\infty^2/2$ –mean pressure in an elementary volume of liquid; α –rarefaction coefficient; ρ –liquid density; u_∞ –velocity of the liquid at infinity

In what follows, to simplify the problem, we shall assume that in the liquid under consideration the density of nuclei is large, that the liquid is homogeneous in strength, and that any volume of it can be characterized by a critical pressure P_k, upon attainment of which a cavitation cavity is formed in the liquid. Such a cavitation cavity may also consist of several gas or vapor bubbles, the dynamics of which is associated with one and the same pressure drop. We shall also assume that the air bubbles that have formed do not exert any substantial influence on the characteristics of the turbulent flow.

Taking into account the assumptions made, we formulate the problem of the frequency of formation of cavitation cavities as follows: it is required to determine the frequency of formation of cavitation cavities in a certain volume of turbulent flow Δv_i, equal to the correlation volume of the flow, in the case when this frequency coincides with the frequency of pressure decrease to the level of the critical pressure P_k. The adopted notations for pressures in the investigated volume of turbulent flow are explained by Fig. 1. Formu-

problem is analogous to the problem solved by Rice ([4]) for the frequency N_i of exceedances by spikes of electrical noise with amplitudes A, normally distributed, above a certain fixed level A_k. The result obtained by Rice can be represented in the form

$$N_i = \frac{1}{2\pi} \exp\left[-\frac{(A_k - \overline{A})^2}{2\sigma^2}\right] \left\{\int_0^\infty \Phi(\omega)\,\omega^2 d\omega \bigg/ \int_0^\infty \Phi(\omega)\,d\omega - \overline{A}^2\right\}^{1/2}, \quad (1)$$

where ω is the circular frequency; $\Phi(\omega)$ is the spectral density of the random variable A; σ is the root-mean-square deviation; $\overline{A}$ is the mean value of the random variable A. The expression in braces in formula (1) represents the root-mean-square circular frequency of the random process.

Since the pressure pulsations in a turbulent flow may be regarded as normally distributed ([5]):

$$W(P) = \frac{1}{\sigma(\mathrm{Re})\sqrt{2\pi}} \exp\left[-\frac{(P - \overline{P})^2}{2\sigma^2}\right];$$

$$\sigma = \begin{cases} 0, & \mathrm{Re} < \mathrm{Re}_1 = u_1 l/\nu, \\ kp(u_\infty - u_1)^2, & \mathrm{Re} > \mathrm{Re}_1, \end{cases} \quad (2)$$

where u_1 is the critical velocity (for $u > u_1$ the flow is turbulent); k is a constant; l is the characteristic size of the body; ν is the kinematic viscosity; Re is the Reynolds number, formula (1) can be used to calculate the frequency of formation of cavitation cavities in a volume equal to the correlation volume. In order to find the total number of cavitation cavities formed near the body being flowed around during 1 sec., it is necessary to sum the values of N_i over all elementary volumes Δv_i, taking into account that the values of the mean rarefaction $\overline{P_i} = P_0 - \alpha\rho u_\infty^2/2$ and of the root-mean-square deviation $\sigma_i(\mathrm{Re})$ depend on the location of the elementary volume Δv_i:

$$N_v = \sum_{n=i}^{v/v_i} \frac{1}{2\pi} \exp\left[-\frac{(P_k - \overline{P_i})^2}{2\sigma_i^2}\right] \left\{\int_0^\infty \Phi_i(\omega)\,\omega^2 d\omega \bigg/ \int_0^\infty \Phi_i(\omega)\,d\omega - \overline{P_i}^2\right\}^{1/2}. \quad (3)$$

Formula (1) can be checked by comparing it with the experimental data of ([1]). In that work, a cavitation cavity is understood as an accumulation of gas bubbles in a vortex detaching from a circular cylinder. Therefore let us define the root-mean-square frequency of the pressure pulsations in formula (1) as the root-mean-square frequency of formation of vortices near the cylinder, neglecting small-scale pulsations. It is known that at $\mathrm{Re} = 10^3 \div 10^5$ a Bénard –Kármán vortex street is formed behind a circular cylinder, and the process of vortex formation at a constant flow velocity is close to periodic; as a result the root-mean-square frequency of the process is close to the mean, and the dependence of the root-mean-square frequency on the flow velocity at infinity u_∞ and the cylinder diameter l can be represented in the form

$$\sqrt{\overline{\omega^2}} \simeq 2\pi(0.18 \div 0.22)\, u_\infty/l, \tag{4}$$

where $0.18{\div}0.22$ is the Strouhal number St for vortices detaching from a circular cylinder ([6]).

Substituting into formula (1) the expressions for the mean pressure in the selected elementary volume and for the root-mean-square pressure deviation

and for the root-mean-square circular frequency, we obtain

$$N_i(0.18{\div}0.22)\frac{u_\infty}{l}\exp\left[-\left(\frac{P_0-P_\mathrm{k}}{\rho(u_\infty-u_1)^2}-\frac{au_\infty^2}{2(u_\infty-u_1)^2}\right)^2\Big/2k^2\right] \quad \text{for } \mathrm{Re}>\mathrm{Re}_1\,. \tag{5}$$

For a circular cylinder the critical Reynolds number is $\mathrm{Re}_1 = 50$. The quantity $(P_0 - P_\mathrm{k})/\frac{1}{2}\rho(u_\infty - u_1)^2 = Q$, i.e., the cavitation number. For $u_\infty \gg u_1$, this notation is identical to that adopted in the hydrodynamics literature.

Formula (5) may also be written in another form:

$$N_i = (0.18 \div 0.22)\frac{u_\infty}{l}\exp\left[-\frac{(Q-a)^2}{8k^2}\right] \quad \text{for } \mathrm{Re} > 50,\ u_\infty \gg u_1. \tag{6}$$

From formulas (4) and (5) one can find the dependence of the Strouhal number of cavitation cavities on the flow parameters and on the shape of the body being flowed around:

$$\mathrm{St} = \frac{Nl}{u_\infty} = (0.18 \div 0.22)\exp\left[-\frac{(Q-a)^2}{8k^2}\right] \quad \text{for } \mathrm{Re} > 50,\ u_\infty \gg u_1. \tag{7}$$

The formulas (6) and (7) include the constants a and k. We determine the value of the rarefaction coefficient a for the point of vortex formation (the point of separation of the turbulent boundary layer from the wall of a circular cylinder) from the experimentally found ([7]) pressure distribution around a circular cylinder at $\mathrm{Re} = 10^5 \div 10^6$: $a = 2.00 \div 2.1$. The value of the proportionality

Fig. 2. Dependence of the number of cavitation cavities N and the Strouhal number of cavitation cavities St on the flow velocity v at constant cavitation number Q. Points are experimental data from work ([1]). Lines are theoretically found values of N and St for cavitation cavities for two values of the Strouhal number of vortices, 0.18 (1) and 0.22 (2).

Figure 1: Fig. 2. Dependence of the number of cavitation cavities N and the Strouhal number of cavitation cavities St on the flow velocity v at constant cavitation number Q. Points are experimental data from work ([1]). Lines are theoretically found values of N and St for cavitation cavities for two values of the Strouhal number of vortices, 0.18 (1) and 0.22 (2).

coefficient between the root-mean-square deviation of the pressure and the flow velocity in the stream, k, was taken equal to 0.58. It was obtained by Batchelor ([5]) for the case of isotropic turbulence. It is obvious that the value of the root-mean-square pressure deviation in the boundary layer will be ≥ 0.58.

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The formulas (6) and (7) include the flow velocity at infinity in an unbounded medium. The experiments of K. K. Shalnev were carried out using a hydrodynamic tunnel; therefore, when comparing theoretical and experimental data it is necessary to take into account the correction for the influence of the walls. Since the flow velocity u in work ([1]) is defined as the mean velocity for the entire live cross section of the hydrodynamic tunnel in the absence of the model, then in calculating the frequency of formation of cavitation cavities by formula (6) one must take $u_\infty = 1.17u$, and in calculating the Strouhal number of cavitation cavities by formula (7), $u_\infty = 1.26u$. These corrections were determined experimentally by K. K. Shalnev.

In Fig. 2 the results of calculating the frequency of formation of cavitation cavities and their St number are compared with the experimental data of work ([1]). The dependence of St of cavitation cavities on the cavitation number Q is given in Fig. 3. The number of cavitation cavities was calculated by formula (7).

Usually in hydrodynamics the cavitation number is used to model cavitation phenomena. Formulas (1)–(7) show that maintaining a constant cavitation number does not yet ensure similarity of the frequencies of formation of cavitation cavities, since their number also depends on the size of the body being flowed around and, through the coefficients of the root-mean-square dev–

of the pressure deviation, the mean rarefaction, and the root-mean-square frequency of pulsations–on the Reynolds number. Thus, in order to achieve simi-

Fig. 3. Dependence of the number St of cavitation cavities on the cavitation number Q.

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larity of the frequencies of formation of cavitation cavities, the constancy of the cavitation number, the Reynolds number, and the Strouhal number must be maintained.

Let us consider the scale effect observed when modeling, by the cavitation number, the critical speed of cavitation inception. If the critical cavitation speed is taken to be the speed beginning from which the number of cavitation cavities sharply increases, then it can be determined from the relation

$$\left\{\frac{P_0 - P_\kappa}{\rho(u_\kappa - u_1)^2} - \frac{\alpha u_\kappa^2}{2(u_\kappa - u_1)^2}\right\}^2 = 2k^2 a^2, \tag{8}$$

where a is a certain arbitrarily chosen constant. The formula determining u_κ is written in the form

$$u_\kappa = \frac{1.4kau_1(\mathrm{Re})}{\alpha(\mathrm{Re})/2 + 1.4ka} + \left\{\frac{P_0 - P_\kappa}{\rho\left(\alpha(\mathrm{Re})/2 + 1.4ka\right)} - \frac{0.7ka\alpha u_1^2(\mathrm{Re})}{\left(\alpha(\mathrm{Re})/2 + 1.4ka\right)^2}\right\}^{1/2}, \tag{9}$$

where $u_1 = \mathrm{Re}_1\nu/l$. From formula (9) it may be concluded that an increase in the dimensions of the cylinder leads to a decrease in the critical speed of cavitation inception (scale effect).

Fig. 3. Dependence of the number St of cavitation cavities on the cavitation number Q.
a—experimental data of work ([1]); —absence of cavitation according to the data of work ([7]); —first acoustic signs of cavitation ([7]); —intermittent, visually noticeable cavities ([7]); —jump in resistance to flow ([7]); —separated flow ([7])

Above we considered the application of formula (1) to the case of separated cavitation on a circular cylinder; however, it can also be applied to the case of boundary-layer cavitation. For this it is only necessary to determine properly the root-mean-square frequency of pressure pulsations. Using the data of the Kolmogorov–Obukhov theory of locally isotropic turbulence, it can be shown that the magnitude of the root-mean-square frequency of large-scale pressure pulsations is proportional to u_∞/l. Then, for the frequency of cavity formation in boundary-layer cavitation, the formula

$$N \sim \frac{u_\infty}{l} \exp\left[-\frac{(Q-\alpha)^2}{8k^2}\right]. \tag{10}$$

must hold.

This formula shows that, at constant cavitation number, the frequency of inception of cavitation cavities is proportional to the flow velocity—a conclusion confirmed by Knapp' s experimental work ([2]). A linear dependence of N on the flow velocity can also be observed in the case when the value of the exponent is close to unity, which, for example, is realized when the values of Q and α are small.

In conclusion, we note that the use of the above relations opens up the fundamental possibility of calculating the dependence of the intensity of cavitation noise and cavitation erosion on the characteristics of the flow and of the body being flowed around.

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Note: Figure translations are in progress. See original paper for figures.