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Abstract

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THE PROBLEM OF THE PROPAGATION OF LONG AND SUPERLONG RADIO WAVES AROUND THE EARTH AND THE LOWER IONOSPHERIC LAYERS (C, D and E) IN THE LIGHT OF INFORMATION THEORY

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The solution of the boundary-value problem (1), (2) gives a functional relation between the field and the medium:

$$\{\Phi_i = f_i(\{a_l^{\mathrm{c}}\}_1^N;\ \{\overline{a_l^{\mathrm{nc}}}\}_{N+1}^\infty)\}_{i=1}^N; \quad \{a_l^{\mathrm{c}} = f_l^{-1}(\{\Phi_i\}_1^N;\ \{\overline{a_l^{\mathrm{nc}}}\}_{N+1}^\infty)\}_{l=1}^N, \qquad (1)$$

where $\{\Phi_i\}_1^N$ are parameters determining the essential features of the electromagnetic field for the problem under consideration, while $\{a_l^{\mathrm{c}}\}_1^N$ are essential and $\{a_l^{\mathrm{nc}}\}_{N+1}^\infty$ nonessential parameters of the medium. The separation into $\{a_l^{\mathrm{c}}\}_1^N$ and $\{a_l^{\mathrm{nc}}\}_{N+1}^\infty$ is carried out by introducing ε-nets in the space $H_{\Phi N}$, $\{\Phi_i\}_1^N \in H_{\Phi N}$, where ε determines the accuracy of the idealization.

The problem of reconciling experimental data $\{\Phi_i\}_1^N$ and $\{a_l^{\mathrm{c}}\}_1^N$ at the nodes of the ε-nets consists in imposing the relation (1) on the arguments of the probability distribution function $W_{\mathrm{exp}}[\{\Phi\}^N, \{a^{\mathrm{c}}\}]$. According to information theory ([3]), this leads to a gain of information about the field and the medium. For an approximate reconciliation we apply the method of mixed initial data (abbreviated m.i.d.). From $\{a_l^{\mathrm{c}}\}_1^N$ we single out parameters $\{a_l^{\mathrm{c,d}}\}_1^P$ $(P \leq N)$ that are reliable within the limits of the ε-net, and supplement them with reliable $\{\Phi_i^{\mathrm{d}}\}_1^R$ up to $N = P + R$. Taking them as the initial data of the problem, we find the unknown $\{a_l^{\mathrm{c,x}}\}_{P+1}^N$ from (1):

$$\{\Phi_i^{\mathrm{d}} = f_i(\{a_l^{\mathrm{c,d}}\}_1^P;\ \{a_l^{\mathrm{c,x}}\}_{P+1}^N;\ \{\overline{a_l^{\mathrm{nc}}}\}_{N+1}^\infty)\}_{i=1}^R, \qquad (2)$$

and, substituting $\{a_l^{\mathrm{c,x}}\}_{P+1}^N$ and $\{a_l^{\mathrm{c,d}}\}_1^P$ into the remaining equations (1), we find $\{\Phi_i^{\mathrm{x}}\}_{R+1}^N$. For $f < 100$ kc/s we take as $\{\Phi_i\}_1^N$ the parameters of the normal waves TH_j and TE_k: $\nu_{j,k}$ and $N_{j,k}$, where ν_j and ν_k lie on the initial portions of

Fig. 1

Figure 1: Fig. 1

the branches (see ([1],[2])) $\nu_j(a,c)$ and $\nu_k(a,c)$, bounded from above by the region of merging with the branches $\nu_{j,k}(c,\infty)$.

$$E_r(a,\theta) \cong \sqrt{\frac{\overline{W}}{\sin\theta}}\, e^{i\frac{\pi}{4}} \left\{ \sum_{j=0}^{n} \eta_j^2 e^{i\nu_j\theta} + \sum_{k=1}^{m} \eta_k^2 e^{i\nu_k\theta} \right\} \text{ in } \frac{\mu\text{V}}{\text{m}}, \tag{3}$$

where $\eta_{j,k} = 0.1829 \cdot 10^{-20}\, \nu_{j,k}^{3/4} \lambda(\pi/N_{j,k})^{1/2}$, where λ is the wavelength in kilometers, W is the radiated power in kilowatts. (5) are Dirichlet polynomials ([4]); from a given function $E_r(a,\theta)$ one can calculate $\nu_{j,k}$ and $\eta_{j,k}$. From the solution of the direct problem ([1],[2]) with $\|\varepsilon(r)\|$, arbitrary within the regularity conditions on the nonessential intervals $(0, a-\delta_1)$ and $(a+h_0+\delta_2, \infty)$, and from the solution of the inverse problem by methods ([5],[6]), which reconstruct $\|\varepsilon(r)\|$ from prescribed $\{\Phi_i\}_1^N$, supplemented arbitrarily to an infinite spectrum under the conditions $\lim \nu_{j,k} \to \infty$ and $\lim_{k,j\to\infty} j/\nu_{j,k} = \sigma$, it follows that between $\{\Phi_i\}_1^N$ and $\|\varepsilon(r)\|$ in $(a-\delta_1,\ a+h_0+\delta_2)$, within the accuracy of the ε-net, there exists a one-to-one correspondence. For $f < 100$ kc/s under terrestrial conditions ($a = 6370$ km), $\delta_1 = 0.01$-0.02 km, $h_0 = 55$ km, $\delta_2 = 25$ km (by day), and $h_0 = 85$ km, $\delta_2 = 15$ km (by night). It is known from experiment that $\|\varepsilon_0\| = \varepsilon_0 + i\,4\pi\sigma_0/\omega$ in the layer $(a-\delta_1, a)$. In the layer (a,c) we put $\|\varepsilon_1\| = 1$. In the layer $(c, c+\delta_2)$, $\|\varepsilon_2\|$ depends on H_0, ν_{eff},

and N_e ([1]). From experiment $H_0 = 0.5$ gauss, while from pressure and temperature data, according to Nicolet ([7]),

$$\nu_{\text{eff}} = 9.5\, e^{-0.148(h-89)} \pm 5\%,$$

where $h = r - a$. The information on N_e is very scanty; therefore we regard $N_e(r)$ as unknown. In the layers $(0, a-\delta_1)$, which are unimportant for the present problem, we set

Fig. 1

$$\|\varepsilon\| = \varepsilon_0' + i\,4\pi\sigma_0/\omega,$$

and in $(c+\delta_2, \infty)$ $H_0 = 0.5$ gauss, ν_{eff} according to Nicolet, and $N_e = N_e(r = c+\delta_2)$. To determine $N_e(r)$ in $(c, c+\delta_2)$, proceed as follows. We divide it into layers (r_l, r_{l+1}), $l = 1, 2, \ldots, R$, and in them replace $N_e(r)$ by segments of straight lines or exponentials with arbitrary parameters $\{a_l^{c,x}\}_1^S$. The approximating function $\widetilde{N}_e[\{a_l^{c,x}\}_1^S]$ will be considered continuous and equal to 0 at $r = c$, where c is unknown. Then the number S of parameters $a_l^{c,x}$ will be equal to

Fig. 2

Figure 2: Fig. 2

the number of (r_l, r_{l+1}). Numerical experiments have established that the mean length of the interval (r_l, r_{l+1}) is 1-2 km and $R = 10$-20. From ([8–10]) we were able to obtain $\nu_{j,k}$ and $n_{j,k}$ only for the first types of normal waves, and the system of equations (2) was incomplete $(R + P < N)$. Using the equivalence of the ray and spectral representations ([12]), i.e., of the systems $\{\nu_{j,k}; n_{j,k}\}$ and $\{\|R_\parallel(\varphi)\|; \|R_\perp(\varphi)\|\}$ $(0 \leq \varphi \leq 90)$, where $\|R_\parallel$ and $\|R_\perp\|$ are the reflection coefficients from the ionosphere, and φ is the angle of incidence at $r = c$, we supplemented (2) with equations of the form

$$\|R_\parallel\| = f_\parallel(\varphi; N_e(r))$$

and

$$\|R_\perp\| = f_\perp(\varphi; N_e(r)).$$

Fig. 2

The numerical values of $\|R_\parallel\|$ and $\|R_\perp\|$ were taken from ([11]). In all equations (2), $\{a_l^{c,x}\}_1^R$ enter through impedance functions $((^2), (11))$, calculated by S. P. Lomnev on the BESM computer of the Computing Center of the Academy of Sciences of the USSR. The functions $\widetilde{N}_e(r)$ constructed from the found $\{a_l^{c,x}\}_1^R$ are given in Fig. 1 for a summer day at middle latitudes $(N_e^{\mathrm{d}}(r))$ and for the night 5-6 hours after sunset

Fig. 3. Parameters of the normal waves of quasi TH_j $(j = 0, 1, 2, 3, 4, 5)$ for daytime. The vertical straight strokes are data for telegraph stations; the wavy ones are for atmospheric discharges. **A** and **B**—attenuation coefficients β in phenomenological formulas for calculating the effective field value

$$E = \frac{3 \cdot 10^5}{a\theta} \sqrt{W_{\mathrm{kV}}}\, e^{-\beta\theta} \quad (\text{in } \mu\text{V/m}),$$

where $a = 6370$ km, θ—angular scattering:
A—$\beta = 31.8/\lambda^{1.25}$ (according to Espenschied, Anderson, and Bailey);
B—$\beta = 8.0/\lambda^{0.6}$ (according to Austin);
C—$\beta_0 = \dfrac{a}{2h}\sqrt{\dfrac{\pi c}{\lambda\sigma}}$ (TEH);
D—$\beta_0^{\mathrm{diff}} = -23.4/\lambda^{1/3}$;
E—$\beta_1^{\mathrm{diff}} = -76/\lambda^{1/3}$;
F—$\beta_2^{\mathrm{diff}} = -117/\lambda^{1/3}$;
β_0^{diff}—attenuation coefficients in the absence of the ionosphere.

$(N_e^H(r))$.

They lie within the "accuracy tracks" caused by the finiteness of the network and by dispersion of the data.

Because of the one-to-one correspondence of the networks $\{\Phi_i\}^N$ and $\|\varepsilon\|$, the exact solution is unique and lies inside the track. The tail $\widetilde{N}_e^D(r)$ $(57 < n < 67$ km) provides the observed attenuation of the far field for $10 < f < 30$ kHz. It does not depend on the solar angle χ, whereas the middle of $\widetilde{N}_e^D$ shifts as $\log\sec\chi$, increases with latitude λ, and in winter, when $\lambda > 50°$, the middle of $\widetilde{N}_e^D$ rises, forming a "table" at $h = 70$–75 km. This increases the attenuation β_{TH_1} to 2.5-3. The "tail" appears in the morning at $\chi = 98°$, moving downward, as shown in Fig. 1, together with the first rays of the sun (which do not contain ultraviolet, but are capable of destroying O_2^-). Ta-

Thus, it is an independent stable formation of the ionosphere, not connected with solar ionization. At the general colloquium of the Computing Center of the Academy of Sciences of the USSR, where this work was reported on March 17, 1960, we proposed calling it the C layer. Very likely, it is produced by cosmic rays, as Nicolet assumes [20]. Then, beyond the line L_α, there remains the explanation of the middle of N_e^d [19], which should be called the D layer. Its maximum (see Fig. 1) is masked by the "tail" of the E layer. The calculation for curve A (Fig. 1) showed large contradictions with the observations of reflection in the region of the upper bend of A at $f > 50$ kHz and $\varphi < 50°$. In Fig. 2 are given $\|R_\parallel(\varphi)\|$ and $\|R_\perp(\varphi)\|$, calculated for $N_e^d(r)$ in the middle of the path at $f = 16$ kHz. The curves for $f = 10$; 25; 50 and 100 kHz are analogous, but with increasing f the reflections fall at steep angles φ to 0.001 at $f = 100$ kHz. The data of Fig. 3 were calculated for the same $N_e^d(r)$. For all TH_j (Fig. 3) there is a transition from waveguide propagation to diffraction propagation with increasing f. It causes a change of the leading waves in (3), explaining the dip in the spectrum of atmospherics at $f = 2 \div 6$ kHz [18] and the abrupt change of the field at $f = 30 \div 35$ kHz. In Fig. 3 are also given calculations of v_j and n_j for the stepwise curve $N_e^d(r)$ (C in Fig. 1). The calculations were made for the cases: a) allowing for sphericity and b) for a flat earth and ionosphere. The former were made in [17] and are marked in Fig. 3 by thin lines, the latter by dashed lines. It is evident that case a) is applicable for $f < f_{\max} = 30$ kHz and $D > D_{\min} = 300$ km, while for case b) $f_{\max}$ falls to 10 kHz. Nevertheless, in [15,16] model b) is applied to explain data on the field at $500 \le f \le 30000$ Hz and $D_{\min} = 50$ km. The inadmissibility of this follows from those very works [15,16]. With increasing f from 500 to 30000 Hz the author is forced to change the parameter $a^{c,x}$; $h = c - a$ from 70 to 50 km, and N_e^0/ν_{eff}^0 from $7 \cdot 10^{-5}$ to $5 \cdot 10^{-7}$. This means that system (2) is inconsistent, and idealization b) is inadmissible. To explain the transition of the linear phase velocity v_j through c in case b), one has to assume that $N_e < 0$!

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Note: Figure translations are in progress. See original paper for figures.