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Abstract

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ON THE CONFIGURATION OF CONGRUENCES H

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In the proposed note a pair of congruences is considered, characterized by the focality of all its submanifolds, and a Möbius configuration is constructed from such pairs of congruences.

1. Let the lines M_1M_2 and M_3M_4 describe a focal pair of ruled surfaces ([1]), and let M_1, M_2, M_3, M_4 be analytic points—the vertices of a canonical frame, whose derivation formulas have the form

$$dM_1 = (xM_1 + \alpha M_2 + M_3)\omega_1^3, \qquad dM_2 = (\alpha M_1 + yM_2 + M_4)\omega_1^3,$$

$$dM_3 = (M_1 + zM_3 + \alpha M_4)\omega_1^3, \qquad dM_4 = (M_2 + \alpha M_3 + tM_4)\omega_1^3,$$

where $x + y + z + t = 0$.

The vertices of the frame are P-points, i.e. the points of intersection of the directrices of the osculating (i.e. having second-order contact with each of the ruled surfaces of the focal pair) special linear complexes with the corresponding rays of the pair. As for the geometric meaning of the invariants of the frame, it can be established analogously to the way this is done, for example, in ([3]).

Let us note one special class of focal pairs of ruled surfaces, determined by the natural equation $(x + y)(x + z) = 0$. This pair is characterized by the fact that the ruled surfaces $\{M_1M_3\}$ and $\{M_2M_4\}$ form a focal pair. In this case we shall say of the original pair that it admits the transformation PD. The class of pairs of ruled surfaces under consideration is further characterized by the fact that the pair $\{M_1M_3\}$ and $\{M_2M_4\}$ admits the transformation PD, and the transformed pair coincides with the original one.

2. A pair of congruences all pairs of whose corresponding ruled surfaces are focal will be called a pair P. In terms of the torsal frame (([2]), Chap. XIII), a pair P is defined by the relations $\omega_1^4 = \omega_4^1 = \omega_2^1 = \omega_3^2 = 0$, $a = b - b' = c = 0$, and is characterized by the fact that its arbitrary submanifold admits the transformation PD.

The pairs H ([3], [4]) are a special case of pairs P and are distinguished from the latter by the conditions

$$\beta = \beta' = 0.$$

3. In Chap. XVI of the monograph ([2]) a construction of the Möbius configuration for congruences W is given. However, this class does not exhaust all congruences admitting the construction of the mentioned configuration.

Let now the congruences $\{A_1 A_2\}$ and $\{A_3 A_4\}$ form a pair P, and let $\{A_3' A_4'\}$ be an arbitrary transformation T of the congruence $\{A_1 A_2\}$, and $dA_i = \omega_i^k A_k$ $(i, k = 1, 2, 3, 4)$, where $A_1' = A_1,\ A_2' = A_2,\ A_3' = \xi A_1 + \tilde{\beta} A_2 + A_3$,

$$A_4' = \tilde{\beta}' A_1 + \eta A_2 + A_4,$$

where

$$\begin{aligned}
\tilde{\beta} &= \beta - \bar{\beta}, & d\tilde{\beta} + \xi\omega_1^2 - \eta\omega_3^4 + \tilde{\beta}\left(\omega_2^2 - \omega_3^3 - \xi\omega_1^3 - \eta\omega_2^4\right) = 0,\\
\tilde{\beta}' &= \beta' - \bar{\beta}', & d\tilde{\beta}' - \xi\omega_4^3 + \eta\omega_2^1 + \tilde{\beta}'\left(\omega_1^1 - \omega_4^4 - \xi\omega_1^3 - \eta\omega_2^4\right) = 0,\\
\bar{\omega}_3^1 - \omega_3^1 &= d\xi + \xi\left(\omega_1^1 - \omega_3^3\right) + \tilde{\beta}\omega_2^1 - \tilde{\beta}'\omega_3^4 - \xi^2\omega_1^3 - \tilde{\beta}\tilde{\beta}'\omega_2^4,\\
\bar{\omega}_4^2 - \omega_4^2 &= d\eta + \eta\left(\omega_2^2 - \omega_4^4\right) + \tilde{\beta}'\omega_1^2 - \tilde{\beta}\omega_4^3 - \eta^2\omega_2^4 - \tilde{\beta}\tilde{\beta}'\omega_1^3.
\end{aligned} \tag{*}$$

Requiring that there exist a congruence $\{B_1 B_2\}$ which would be a transformation T both for the congruence $\{A_3 A_4\}$ and for the congruence $\{A_3' A_4'\}$, we obtain

$$(\rho - b)(\bar{b} - b') = 0. \tag{**}$$

If $\rho = b$, then from the totality of congruences $\{B_1 B_2\}$ one is distinguished which forms a pair P with the congruence $\{A_3' A_4'\}$.

Let now $\bar{b} - b' = 0$. Then the congruences $\{A_1 A_2\}$ and $\{A_3' A_4'\}$ form a pair P (under the condition that $\{A_1 A_2\}$ is not a congruence W). In this case the congruences $\{A_3 A_4\}$ and $\{B_1 B_2\}$, $\{A_3' A_4'\}$ and $\{B_1 B_2\}$ form pairs P.

If, however, both factors of equation $(**)$ vanish and the congruences $\{A_1 A_2\}$ and $\{A_3 A_4\}$ form a pair H, then from equations $(*)$ it follows that

$$d(\Delta + \bar{b}) = (\Delta + \bar{b})\, db,$$

where $\Delta^2 = (A_3' A_4' B_1 B_2)$. Setting $\Delta + \bar{b} = 0$, we obtain that the congruences $\{A_3' A_4'\}$ and $\{B_1 B_2\}$ form a pair H.

Thus we have proved that there exists a Möbius configuration consisting of four congruences H: $\{A_1A_2\}$, $\{A_3A_4\}$, $\{A_3'A_4'\}$, and $\{B_1B_2\}$, with the congruences $\{A_1A_2\}$ and $\{A_3A_4\}$, $\{A_3'A_4'\}$ and $\{B_1B_2\}$ forming pairs H.

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CITED LITERATURE

[1] S. P. Finikov, *Izv. Acad. Sci. USSR, Ser. Math.*, **9**, 79 (1945). [2] S. P. Finikov, *Theory of Pairs of Congruences*, Moscow, 1956. [3] R. N. Shcherbakov, *Mat. Sbornik*, **46** (88), 2, 159 (1958). [4] M. B. Pergamenshchikov, Reports of the Scientific Conference on Theoretical and Applied Problems of Mathematics and Mechanics, Tomsk, 1960, p. 74.

Note: Figure translations are in progress. See original paper for figures.