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1961

Abstract

Full Text

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DISPERSION RELATIONS FOR THE COMPTON EFFECT ON NUCLEONS

(Presented by Academician N. N. Bogolyubov, 1 IV 1961)

In this paper, by the method proposed in ([1]), dispersion relations are obtained for the "physical" amplitudes of the process of photon scattering by nucleons (12). By unitarity in the one-meson approximation ([2]), the dispersion integrals can be calculated if photoproduction data are used.

1. The kinematics of processes with two photons has been analyzed in detail in ([3]), and our derivation is based on its results. The matrix element of the process is written as follows.

$$\langle f|\hat{S}-1|i\rangle = \frac{(2\pi)^4}{i}\,\frac{\delta(p+k-p'-k')}{\sqrt{4k_0k_0'p_0p_0'}}\sum_{i=1}^{6}\sum_{\mu,\nu=0}^{3}\left[-\Omega_i(\nu,\nu_1)\,\bar{u}_{p'}(e'_\mu\hat{R}^i_{\mu\nu}e_\nu)u_p\right]; \tag{1}$$

$$\nu = k\cdot(p+p') = k'\cdot(p+p'); \qquad \nu_1 = k\cdot k'; \qquad \hbar = c = M = 1;$$

$$\bar{u}_p u_p = 1; \qquad (a\cdot b) = a_0b_0 - \mathbf{ab}.$$

(For the notation see ([2,3]).) $\Omega_i(\nu,\nu_1)$ are unknown scalar functions possessing an isotopic structure:

$$\Omega_i(\nu,\nu_1) = \tau_p\Omega_i^{(p)} + \tau_n\Omega_i^{(n)} = \frac{1+\tau_3}{2}\Omega_i^{(p)} + \frac{1-\tau_3}{2}\Omega_i^{(n)}.$$

The form and number of $\hat{R}^j_{\mu\nu}$ are determined by the conservation laws $p+k = p'+k'$, by the equations of motion $\hat{p}u_p = u_p$; $\bar{u}_p\hat{p} = \bar{u}_p$, by relativistic invariance, gauge invariance ($(k_\mu R_{\mu\nu}e_\nu) = (e'_\mu R_{\mu\nu}k_\nu) = 0$), invariance under weak (Wigner) time reversal, $ek = e'k' = 0$.

It is convenient for us to choose the following 6 linearly independent operators

$$\hat{R}^i = (e'_\mu\hat{R}^i_{\mu\nu}e_\nu), \qquad \hat{R}^1 = e\cdot e' - \frac{e'\cdot K\, e\cdot K}{\nu_1};$$

$$\hat{R}^2 = \left(e\cdot e' - \frac{e'\cdot K\, e\cdot K}{\nu_1}\right)\frac{\hat{K}}{2\nu};$$

$$\hat{R}^3 = e'\cdot Le\cdot L\,\frac{\nu_1}{\nu^2}; \qquad \hat{R}^4 = e'\cdot Le\cdot L\,\frac{\nu_1}{\nu^2}\frac{\hat{K}}{2\nu}; \tag{2}$$

$$\hat{R}^5 = \left[\hat{e}'\hat{e} - \frac{(\nu+\nu_1)e\cdot e' + e'\cdot Pe\cdot P - e'\cdot Ke\cdot K}{\nu}\right]\hat{K} + \hat{e}'\hat{e}\,(P-K) + \hat{e}\hat{e}'\,(P+K);$$

$$\hat{R}^6 = \left[\hat{e}'\hat{e} - \frac{2\nu_1 e\cdot e' + e'\cdot Pe\cdot K + e'\cdot Ke\cdot P}{2\nu_1}\right]\hat{K} + 2e\cdot e' - 2\frac{e'\cdot Ke\cdot K}{\nu_1} +$$

$$+\hat{e}'e\cdot K\,\frac{\nu-\nu_1}{\nu_1} + \hat{e}e'\cdot K\,\frac{\nu+\nu_1}{\nu_1}.$$

They are related linearly to $\hat{G}^{\pm}_{j(\mu,\nu)}$ of the paper ([3]):

$$\hat{R}^1 = \hat{G}^+_1, \qquad \hat{R}^2 = \frac{1}{2\nu}\hat{G}^-_1; \qquad \hat{R}^3 = \frac{\nu_1}{\nu^2}\hat{G}^+_2; \qquad \hat{R}^4 = \frac{\nu_1}{\nu^2}\frac{\hat{G}^-_2}{2\nu};$$

$$\hat{R}^5 = \hat{G}^+_9 + \hat{G}^+_{10} - \frac{1}{\nu}\left(\nu_1\hat{G}^-_1 + \hat{G}^-_2\right); \qquad \hat{R}^6 = \hat{G}^+_{10} + 2\hat{G}^+_1; \tag{3}$$

$$K = k + k'; \qquad P = p + p'; \qquad L = P - \frac{\nu}{\nu_1}K; \qquad \hat{K} = \gamma_0 K_0 - \vec{\gamma}\,\vec{K}$$

(instead of $\hat{G}^+_5$ ([3]), it is convenient to take $\hat{G}^+_{10}$ ([3])). The invariant factors ν_1, ν have been introduced so that the $\hat{R}^j$ are symmetric under a transformation of the "crossing-symmetry type" ([3]):

$$(\hat{R}^i)_{\tilde{k}} \equiv \left[R^i(p, s \leftrightarrow p', s';\ e \leftrightarrow e';\ k, k' \to -k, -k')\right]^+ = \hat{R}^i; \tag{4}$$

then

$$\Omega_i(\nu,\nu_1) = \Omega^*_i(-\nu,\nu_1) \quad \text{or}$$
$$\operatorname{Re}\Omega_i(\nu,\nu_1) = \operatorname{Re}\Omega_i(-\nu,\nu_1); \quad \operatorname{Im}\Omega_i(\nu,\nu_1) = -\operatorname{Im}\Omega_i(-\nu,\nu_1). \tag{5}$$

In the center-of-inertia system the amplitude of the process $\hat{\Phi} = \hat{\Phi}(\omega,\theta)$ $\left(\hat{\Phi} = \frac{1}{\omega}\hat{R};\ \theta\right.$ is the angle; ω is the total energy in the center-of-inertia system, $\mathbf{n} = \mathbf{k}/k_0,\ \mathbf{n}' = \mathbf{k}'/k_0)$ can be represented in the form

$$\hat{\Phi} = \Phi_j(\omega,\theta) n_j = \Phi_1\, \mathbf{e}\cdot\mathbf{e}' + \Phi_2\, \mathbf{n}'\cdot\mathbf{e}\,\mathbf{n}\cdot\mathbf{e}' + \Phi_3\, \vec{\sigma}\cdot\mathbf{n}'\,\vec{\sigma}\cdot\mathbf{n}\,\mathbf{e}\cdot\mathbf{e}' + \Phi_4\, \vec{\sigma}\cdot\mathbf{n}'\,\vec{\sigma}\cdot\mathbf{n}\,\mathbf{e}'\cdot\mathbf{n}\,\mathbf{e}\cdot\mathbf{n}' + \Phi_5\, \vec{\sigma}\mathbf{e}'\,\vec{\sigma}\mathbf{e} + \Phi_6\, \vec{\sigma}\cdot(\mathbf{e}'\times\mathbf{n}')\,\vec{\sigma}\cdot(\mathbf{e}\times\mathbf{n}). \quad (6)$$

Using the concrete representation of $\bar{u}_{p'}$ and u_p in the center-of-inertia system, we easily obtain:

$$R_j = \bar{u}_p\, \hat{R}^j u_p = \sum_k c_{jk} n_k, \quad (7)$$

It is not difficult to obtain the inverse formulas as well:

$$n_i = \sum_k (c_{ik})^{-1} R_k \quad (8)$$

The matrices c_{ik} and $(c_{ik})^{-1}$ are not given for lack of space.

Finally, let us give the expression of $\Phi_j(\omega,\theta)$ in terms of Legendre polynomials and the transition probabilities into states with definite total angular momentum $I = L + s = L' + s'$, and with parities of the initial and final photon Π_L and $\Pi_{L'}$, respectively, which is readily obtained with the aid of "generalized phase-shift analysis" :

$$\Phi_1 = \sum_{l=1}^{\infty}\{(xP_l'')R(E) + P_{l+1}''R_+(E) - P_{l-1}''R_-(E) + l^2P_{l+1}'R_+(M)$$
$$- (l+1)^2P_{l-1}'R_-(M) - 2P_l''R(M) + (1-x^2)P_l'''[R(E,M) - R(M,E)]\};$$

$$\Phi_2 = \sum_{l=1}^{\infty}\{-P_l''R(E) + P_{l+1}'''R_+(E) - P_{l-1}'''R_-(E) - (xP_{l+1}'')'R_+(M)$$
$$+ (xP_{l-1}'')'R_-(M) + [(2P_{l+1}''' - (l+3)P_l'')R(E,M)$$
$$- (2P_l'' + (l-1)P_{l+1}'')R(M,E)]\};$$

$$\Phi_3 = \sum_{l=1}^{\infty}\{-2P_l''R(E) + (2P_{l+1}'' - (l+1)(l+2)P_l')R(M)$$
$$+ [l(l+1)P_{l+1}' - 4P_l'']R(E,M) - [(l+1)(l+2)P_l' - 4P_{l+1}'']R(M,E)\};$$

$$\Phi_4 = \sum_{l=1}^{\infty}\{-P_l'''R(E) + (xP_l')''R(M) - [(l+1)P_{l+1}'' - 2P_l'']R(E,M)$$
$$- [(l+1)P_l'' - 2P_{l+1}''']R(M,E)\};$$

$$\Phi_5 = \sum_{l=1}^{\infty}\{P_l''R(M) - (xP_l')'R(E) + [l(l-1)P_l' - 2P_{l-1}'']R(E,M)$$
$$- [l(l+1)P_{l+1}' - 2P_l'']R(M,E)\};$$

$$\Phi_6 = \sum_{l=1}^{\infty}\{P_l''R(E) - (xP_l')'R(M) + [l(l-1)P_l' - 2P_{l-1}'']R(M,E)$$
$$- [l(l+1)P_{l+1}' - 2P_l'']R(E,M)\};$$
(9)

$x = \cos\theta$, P_l' is the derivative of the ordinary Legendre polynomial.

$R(E)$, $R(M)$, $R(E,M), \ldots$ are expressed in the following way in terms of the transition probabilities $S(I, \Pi_L, \Pi_{L'}, L, L')$—the "phases" of the Compton effect:

$$R_\pm(E) = \frac{1}{l(l+1)}S(l \pm {}^1/_2,\ (-1)^l,\ (-1)^l,\ l,\ l); \quad R(E) = R_+(E) - R_-(E);$$
$$R_\pm(M) = \frac{1}{l(l+1)}S(l \pm {}^1/_2,\ (-1)^{l+1},\ (-1)^{l+1},\ l,\ l);$$
$$R(M) = R_+(M) - R_-M;$$
$$R(M,E) = \frac{1}{(l+1)\sqrt{l(l+2)}}S(l + {}^1/_2,\ (-1)^l,\ (-1)^{l+1},\ l,\ l+1);$$
$$R(E,M) = \frac{1}{(l+1)\sqrt{l(l+2)}}S(l + {}^1/_2,\ (-1)^{l+1},\ (-1)^l,\ l,\ l+1).$$
(10)

2. In order to apply Cauchy' s theorem to the invariant scalar functions $\Omega_j(\nu,\nu_1)$, it is necessary to know their behavior as $\nu \to \infty$. Let us assume that $\Omega_{1,2,3,4,6}/(\nu-\nu_0)^2$; $\Omega_5/(\nu-\nu_0)$ are analytic functions throughout the complex half-plane ν (apart from the residues defined in (4)). This can be justified in some way by analyzing the behavior of $\hat{R}^j$ as $\nu \to \infty$ and by postulating that the cross sections independent of spin and dependent on spin tend to constants as $\nu \to \infty$. Because of the "crossing symmetry" of the original $\Omega_j(\nu,\nu_1)$, (5), (4), the dispersion relations for $\Omega_j(\nu,\nu_1)$ must contain 6 unknown constants Ω_j^0. However, using the known behavior of the process amplitude at small energies (5), it is easy to determine that $\Omega_j^0 = 0$. Let us note that in (5) the form of the amplitude at small γ-quantum energies is connected with relativistic invariance and gradient invariance, i.e., we may say that the indicated invariances lead to the determination of the unknown "subtraction" constants. This is to some extent analogous to the situation in photoproduction (6).

As a result,

$$\operatorname{Re}\Omega_j(\nu,\nu_1) = \Omega_j^B(\nu,\nu_1) + \frac{2\nu^2}{\pi}\int_{(1+\mu)^2-1-\nu_1}^{\infty} \frac{\operatorname{Im}\Omega(\nu',\nu_1)}{\nu'(\nu'^2-\nu^2)}\,d\nu'; \qquad \mu \simeq \frac{1}{7}; \quad \nu_0 = 0;$$

$$\Omega_1^B = -r_0\{4\nu_1 f\tau_p + (1+4\nu_1 f)\lambda\tau_p\};$$

$$\Omega_2^B = -r_0\{2(\nu_1^2+\nu^2)f\tau_p + 2(\nu_1^2+2\nu^2)f\lambda\tau_p - \frac{\lambda^2}{2}(\nu_1 - 4\nu^2 f)\};$$

$$\Omega_3^B = +r_0\nu^2 f\lambda\tau_p;$$

$$\Omega_4^B = -r_0\{2\nu^2 f\tau_p + 2\nu^2 f\lambda\tau_p - \frac{\nu^2\nu_1}{2}f\lambda^2\}; \qquad f = \frac{1}{\nu_1^2-\nu^2};$$

$$\Omega_5^B = -r_0\{\nu_1 f\tau_p + \nu_1 f\lambda\tau_p - \frac{\lambda^2}{4}\}; \qquad r_0 = e_0^2 \quad (\hbar = c = M = 1);$$

$$\Omega_6^B = -r_0\{-2\nu_1 f\tau_p + \frac{1}{2}(1-4\nu_1 f)\lambda\tau_p + \frac{\lambda^2}{2}\}; \qquad \lambda = 1.79\,\tau_p - 1.91\,\tau_n. \tag{11}$$

3. Formulas (7) and (8) make it possible without difficulty to obtain dispersion relations for $\Phi_j(\omega,\nu_1)$:

$$\operatorname{Re}\Phi_j(\omega,\nu_1) = \Phi_j^B(\omega,\nu_1) + \int_{1+\mu}^{\infty} \varphi(\alpha,\omega,\nu_1)\, I_j(\alpha,\omega,\nu_1)\, d\alpha;$$
$$J_{\left[\frac{1}{3}\right]} = \sum_{1,3} f_{\left[\frac{i}{3}\right]} \operatorname{Im}\Phi_i + \left(\frac{1-\omega}{\pm 1-\omega}\right)(f_1 \operatorname{Im}\Phi_5 + f_3 \operatorname{Im}\Phi_6);$$
$$J_{\left[\frac{2}{4}\right]} = \sum_{1,3} f_{\left[\frac{i}{3}\right]} (A \operatorname{Im}\Phi_i + B \operatorname{Im}\Phi_{i+1}) + \left(\frac{1-\omega}{\pm 1-\omega}\right)(f_2 \operatorname{Im}\Phi_5 + f_4 \operatorname{Im}\Phi_6);$$
$$J_{\left[\frac{5}{6}\right]} = \nu \frac{\omega-1}{\omega+1}\frac{\alpha+1}{\alpha-1}\left(\frac{\operatorname{Im}\Phi_{\left[\frac{5}{6}\right]}}{\alpha-\omega}\right) - \left(\frac{\operatorname{Im}\Phi_{\left[\frac{6}{5}\right]}}{\alpha+\omega}\right);$$
$$\varphi(\alpha,\omega,\nu_1) = \frac{2}{\pi}\left(\frac{\alpha}{\omega}\right)^2 \left(\frac{\omega+1}{\alpha+1}\right)^2 \frac{\omega^2-1-\nu_1}{\alpha^2-1-\nu_1}\frac{1}{\alpha^2+\omega^2-2-2\nu_1}; \qquad \Phi_j^B = \frac{1}{\omega} c_{jk}\Omega_k^B;$$
$$f_1^1 = \frac{\alpha\omega-\alpha+\omega-1-\nu_1}{\alpha-\omega}; \qquad f_3^1 = \left(\frac{\omega-1}{\omega+1}\right)^2 \frac{\alpha\omega+\omega+\alpha+1+\nu_1}{\alpha+\omega}. \tag{12}$$

$$f_1^3 = \left(\frac{\alpha+1}{\alpha-1}\right)^2 \frac{\alpha\omega-\omega-\alpha+1+\nu_1}{\alpha+\omega}; \quad f_3^3 = \left(\frac{\omega-1}{\omega+1}\right)^2 \left(\frac{\alpha+1}{\alpha-1}\right)^2 \frac{\alpha\omega+\alpha-\omega-1-\nu_1}{\alpha-\omega};$$

$$A = \left(\frac{\omega^2-1}{\alpha^2-1}\right)^2 \frac{\omega^2-\alpha^2}{4\omega^2}\frac{(\omega^2-1)(\alpha^2-1-\nu_1)+(\alpha^2-1)(\omega^2-1-\nu_1)}{(\omega^2-1-\nu_1)^2};$$

$$f_{\left[\frac{1}{3}\right]} = \left(\frac{\alpha+1}{\alpha\pm 1}\right)\frac{(\alpha\pm 1)(\omega^2-1)\mp 2\nu_1}{(\alpha-1)(\omega+1)}; \quad B = \left[\frac{\alpha}{\omega}\frac{\alpha^2-1-\nu_1}{\omega^2-1-\nu_1}\left(\frac{\omega^2-1}{\alpha^2-1}\right)^2\right]^2;$$

$$f_{\left[\frac{2}{4}\right]} = \left(\frac{\omega+1}{2\omega}\right)^2 \left(\frac{\omega-1}{\alpha-1}\right)^2 \left\{2\frac{1\mp\alpha}{1+\omega} - (\omega-1)\frac{(\omega^2-1)(\alpha^2-1-\nu_1)+(\alpha^2-1)(\omega^2-1-\nu_1)}{(\omega^2-1-\nu_1)^2}\right\};$$

the integrals are understood in the sense of the principal value.

Let us note that the limiting case of forward scattering ($\nu_1 = 0$) is obtained if one takes into account that

$$\hat{\Phi}(\nu_1 = 0) = f_1 \mathbf{e}\cdot\mathbf{e}' + f_2 i\vec{\sigma}\times\mathbf{e}' = (\Phi_1+\Phi_3+\Phi_5+\Phi_6)\mathbf{e}\cdot\mathbf{e}' + (-\Phi_5-\Phi_6) i\vec{\sigma}\times\mathbf{e}'. \tag{13}$$

Using (12), where $\nu_1 = 0$ has been put, and the fact that $f_i = \omega f_i^{\mathrm{L}}$, $i = 1, 2$ (L indicates that the corresponding quantities are taken in the laboratory coordinate system), we immediately obtain:

$$\operatorname{Re} f_1^{\mathrm{L}} = -r_0\tau_p + \frac{2\delta_{\mathrm{L}}^2}{\pi}\int_{\mu}^{\infty}\frac{\operatorname{Im} f_1^{\mathrm{L}}\, d\delta'_{\mathrm{L}}}{\delta'_{\mathrm{L}}(\delta'^2_{\mathrm{L}} - \delta^2_{\mathrm{L}})}, \quad \operatorname{Re} f_2^{\mathrm{L}} = r_0\frac{\delta_{\mathrm{L}}}{2}\lambda^2 + \frac{2\delta^3_{\mathrm{L}}}{\pi}\int_{\mu}^{\infty}\frac{\operatorname{Im} f_2^{\mathrm{L}}\, d\delta'_{\mathrm{L}}}{\delta'^2_{\mathrm{L}}(\delta'^2_{\mathrm{L}} - \delta^2_{\mathrm{L}})}, \tag{14}$$

which exactly coincides with the known result (7). For application of (12) it is convenient to pass from ω, α to a new variable: the energy δ of the incident γ-quantum, and to expand the corresponding expressions in δ, δ'. We give here the expressions in which, in Φ_j^{B}, only terms of order δ have been retained, while in the integrals terms of order 1 have been retained (something like the "static" limit $\mathrm{B}^{(1)}$); they are especially simple:

$$\operatorname{Re}\Phi_1 = \Phi_1^{\mathrm{B}} + \frac{2\delta^2}{\pi}\int_{\delta_0}^{\infty}\frac{dx}{x(x+\delta)}\left[\frac{\operatorname{Im}\Phi_1}{x-\delta} + \frac{\operatorname{Im}\Phi_5}{x} + \frac{\delta x + \nu_1}{\delta x}\,\frac{\operatorname{Im}\Phi_3 + \operatorname{Im}\Phi_6}{x}\right];$$

$$\operatorname{Re}\Phi_2 = \Phi_2^{\mathrm{B}} + \frac{2\delta^4}{\pi}\int_{\delta_0}^{\infty}\frac{dx}{x^3(x+\delta)}\left[\frac{\operatorname{Im}\Phi_2}{x-\delta} + \frac{\operatorname{Im}\Phi_6}{\delta} + \frac{\delta x + \nu_1}{\delta^2}\,\frac{\operatorname{Im}\Phi_4}{\delta}\right]; \tag{15}$$

$$\operatorname{Re}\Phi_{3,5,6} = \Phi_{3,5,6}^{\mathrm{B}} + \frac{2\delta^3}{\pi}\int_{\delta_0}^{\infty}\frac{dx}{x^2(x^2-\delta^2)}\,\operatorname{Im}\Phi_{3,5,6};$$

$$\operatorname{Re}\Phi_4 = \Phi_4^{\mathrm{B}} + \frac{2\delta^5}{\pi}\int_{\delta_0}^{\infty}\frac{dx}{x^4(x^2-\delta^2)}\,\operatorname{Im}\Phi_4;$$

$$\Phi_1^{\mathrm{B}} = -r_0\left[\tau_p(1-\delta) + \delta\frac{\lambda\tau_p}{2} - \delta\frac{(\lambda+\tau_p)^2}{2}\cos\theta\right]; \quad \Phi_2^{\mathrm{B}} = -r_0\delta\,\frac{\lambda^2 + 2\lambda\tau_p - \tau_p}{2};$$

$$\Phi_3^{\mathrm{B}} = -r_0\frac{\delta(\lambda+1)\tau_p}{2}\cos\theta; \quad \Phi_4^{\mathrm{B}} = r_0\frac{\delta(\lambda+1)\tau_p}{2}; \quad \Phi_5^{\mathrm{B}} = r_0\delta\frac{\lambda\tau_p}{2};$$

$$\Phi_6^{\mathrm{B}} = -r_0\frac{\delta}{2}\left[(\lambda+\tau_p)^2 - (\lambda+1)\tau_p\cos\theta\right]; \quad \delta_0 = \mu\,\frac{1+\mu/2}{1+\mu}.$$

I express my gratitude to L. D. Solov' ev for numerous discussions and a number of valuable suggestions.

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Received
17 III 1961

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Note: Figure translations are in progress. See original paper for figures.