THE INFLUENCE OF RANDOM DISTURBANCES ON THE PERIODIC REGIME IN RELAY AUTOMATIC SYSTEMS

`![Fig. 1](attachment:figure-1)`

1961

SovietRxiv

Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

CYBERNETICS AND THE THEORY OF REGULATION

Ya. Z. Tsypkin

THE INFLUENCE OF RANDOM DISTURBANCES ON THE PERIODIC REGIME IN RELAY AUTOMATIC SYSTEMS

(Presented by Academician B. N. Petrov, 12 XI 1960)

Let us consider a relay automatic system consisting of a relay element and an arbitrary continuous part, including both lumped and distributed parameters (Fig. 1). Suppose that in this system there exists a periodic regime of frequency ω_0. This periodic regime may correspond either to forced oscillations caused by an external periodic action of multiple frequency, or, in the absence of an external action, to self-oscillations. The presence of random actions–fluctuations–in the relay element or in the continuous part changes the periodic regime. Let us estimate the change in the periodic regime caused by the action of random disturbances, i.e., let us find the mean value of the square of this change.

Fig. 1

The equation of the relay system may be written in the form (1)

$$L\{x(t)\} = L\{f(t)\} - W(p)L\{\Phi(x(t))\}, \tag{1}$$

where $f(t)$ is the external action; $x(t)$ is the error; $W(p)$ is the transfer function of the linear part; Φ is the characteristic of the relay element, which for simplicity we shall take equal to

$$\Phi(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0, \end{cases} \tag{2}$$

and

$$L\{f(t)\} = \int_{-\infty}^{\infty} e^{-pt} f(t)\, dt.$$

Let $\tilde{f}(t)$ denote the external periodic action, and $\tilde{x}(t)$ the simplest periodic regime occurring in the system. The equation of the periodic regimes is obtained from equation (1) if $f(t)$ and $x(t)$ in it are replaced by $\tilde{f}(t)$ and $\tilde{x}(t)$.

We shall represent random disturbances as an additional external action $\psi(t)$, applied to the input of the relay element. Thus, in the presence of random disturbances,

$$f(t) = f(t) + \psi(t).$$

As a consequence, the periodic regime is disturbed, and the error $x(t)$ will now be equal to

$$x(t) = \tilde{x}(t) + \xi(t),$$

where $\xi(t)$ is the deviation from the periodic regime.

Substituting these values into (1), we obtain

$$L\{\tilde{x}(t) + \xi(t)\} = L\{\tilde{f}(t) + \psi(t)\} - W(p)L\{\Phi(\tilde{x}(t) + \xi(t))\}. \tag{3}$$

Put $\xi(t) = 0$ in equation (3) and subtract this equation of the periodic regime from equation (3) for $\xi(t) \neq 0$. Then we obtain the equation for the deviations in the form:

$$L\{\xi(t)\} = L\{\psi(t)\} - W(p)L\left\{\frac{\Phi(\tilde{x}(t) + \xi(t)) - \Phi(\tilde{x}(t))}{\xi(t)}\,\xi(t)\right\}. \tag{4}$$

If the random disturbances are sufficiently small, the equation takes the form

$$L\{\xi(t)\} = L\{\psi(t)\} - W(p)L\{\Phi'(\tilde{x}(t))\,\xi(t)\}, \tag{5}$$

where Φ' is the derivative characteristic of the relay element, equal to [(1)]

$$\Phi'(\tilde{x}(t)) = 2k_p\delta(\tilde{x}(t)) = \frac{2k_p}{|\dot{\tilde{x}}(\pi/\omega_0)|}\sum_{k=-\infty}^{\infty}\delta\left(t - k\frac{\pi}{\omega_0}\right). \tag{6}$$

Here k_p is the "gain" coefficient of the relay element, and $\delta(t)$ is the impulse function, or Dirac function. Substituting (6) into (5) and noting that

$$L\left\{\delta\left(t - k\frac{\pi}{\omega_0}\right)\right\} = e^{-pk\frac{\pi}{\omega_0}}\xi\left(k\frac{\pi}{\omega_0}\right),$$

we obtain

Fig. 2

Figure 2: Fig. 2

$$\Xi(p) = \Psi(p) - W(p)\frac{2k_p}{|\dot{\tilde{x}}(\pi/\omega_0)|}\,\Xi^*(p), \tag{7}$$

where the following notation has been adopted

$$\Xi(p) = L\{\xi(t)\}, \qquad \Psi(p) = L\{\psi(t)\}$$

for transforms in the sense of the usual two-sided Laplace transform, and

$$\Xi^*(p) = D\left\{\xi\left(k\frac{\pi}{\omega_0}\right)\right\} = \sum_{k=-\infty}^{\infty} e^{-pk\frac{\pi}{\omega_0}}\xi\left(k\frac{\pi}{\omega_0}\right)$$

for the transform in the sense of the discrete two-sided Laplace transform [1].

Fig. 2

Equation (7) corresponds to a sampled-data system consisting of a linear part with transfer function $W(p)$ and the simplest impulse element with gain coefficient

$$\frac{2k_p}{|\dot{\tilde{x}}(\pi/\omega_0)|},$$

at whose input random disturbances act (Fig. 2).

To determine $\Xi^*(p)$ from (7), we use the relation between $\Xi^*(p)$ and $\Xi(p)$ [1]:

$$\Xi^*(p) = \mathfrak{D}\{\Xi(p)\} = \frac{\omega_0}{\pi}\sum_{m=-\infty}^{\infty}\Xi(p + 2\pi jm\omega_0).$$

Subjecting equation (7) to the $\mathcal{D}$-transform and noting that ([2])

$$\mathcal{D}\left\{\Xi^*(p)\,W^*(p)\right\} = \Xi^*(p)\,W^*(p),$$

where $W^*(p) = \mathcal{D}\{W(p)\}$, we finally obtain an equation for $\Xi^*(p)$:

$$\Xi^*(p) = \frac{\Psi^*(p)}{1 + \dfrac{2k_p}{|\dot{x}\,(\pi/\omega_0)|}\,W^*(p)} = K^*(p)\Psi^*(p), \tag{8}$$

where

$$K^*(p) = \frac{1}{1 + \dfrac{2k_p}{|\dot{x}\,(\pi/\omega_0)|} W^*(p)} \tag{9}$$

is the transfer function of the pulse system.

Explicit expressions for $W^*(p)$ are given in ([1],[2]).

Let us note that equation (8) was used as the basis for investigating the stability of periodic regimes ([1]). If the periodic regime is stable, then the poles of $K^*(p)$ have negative real parts.

The investigation of the effects of random disturbances on a periodic regime reduces to the statistical analysis of the pulse system described by equation (8). For this purpose one may use the method set forth in ([2]).

Let the random disturbances be stationary, and let their spectral density and correlation function be, respectively, $S^*_\psi(\omega)$, $R_\psi\left(n\dfrac{\pi}{\omega_0}\right)$. The spectral density and correlation function of the deviation from the periodic regime are found from the known relations ([2]):

$$S^*_\xi(\omega) = |K^*(j\omega)|^2\, S^*_\psi(\omega),$$

$$R_\xi\left(n\frac{\pi}{\omega_0}\right) = \sum_{m=-\infty}^{\infty}\sum_{r=-\infty}^{\infty} k\left(m\frac{\pi}{\omega_0}\right) k\left(r\frac{\pi}{\omega_0}\right) R_\psi\left((n+m-r)\frac{\pi}{\omega_0}\right),$$

where $k\left(n\dfrac{\pi}{\omega_0}\right)$ is the pulse response of the pulse system, equal to the inverse discrete transform of the transfer function:

$$k\left(n\frac{\pi}{\omega_0}\right) = D^{-1}\left\{K^*(p)\right\}.$$

Knowing the expressions $S^*_\xi(\omega)$ and $R_\xi\left(n\dfrac{\pi}{\omega_0}\right)$, we determine the mean value of the square of the deviation from the periodic regime by the formulas ([2])

$$\overline{\xi^2\left(n\frac{\pi}{\omega_0}\right)} = \frac{1}{\pi}\int_0^{\pi} |K^*(j\omega)|^2\, S^*_\psi(\omega)\, d\omega \tag{10}$$

or

$$\overline{\xi^2\left(n\frac{\pi}{\omega_0}\right)} = R_\xi(0) = \sum_{m=-\infty}^{\infty}\sum_{r=-\infty}^{\infty} k\left(m\frac{\pi}{\omega_0}\right) k\left(r\frac{\pi}{\omega_0}\right) R_\xi\left((m-r)\frac{\pi}{\omega_0}\right). \quad (11)$$

The formulas obtained solve the posed problem. Various methods for computing $\overline{\xi^2\left(n\frac{\pi}{\omega_0}\right)}$, both analytic and graphical, are given in ([2]).

Another approach to determining the mean value of the square of the deviation (11), applicable to relay systems with lumped parameters, is described in ([3]).

The approach described above for estimating the change in the periodic regime is applicable in the case of more complicated types of characteristics of the relay element—

of more complex types of periodic regimes. In these cases the problem reduces to a statistical analysis of a pulse system with several nonsynchronously operating pulse elements ([2]).

In an analogous way one can investigate the influence of fluctuations in relay systems of extremal control ([4]).

Institute of Automation and Remote Control
Academy of Sciences of the USSR

Received
10 XI 1960

REFERENCES

[1] Ya. Z. Tsypkin, *Theory of Relay Automatic-Control Systems*, Moscow, 1955, ch. VIII.
[2] Ya. Z. Tsypkin, *Theory of Pulse Systems*, Moscow, 1958, chs. II, III.
[3] V. Ya. Katkovnik, A. A. Pervozvanskii, *Avtomatika i telemekh.*, **22**, No. 5 (1961).
[4] I. S. Morosanov, *Avtomatika i telemekh.*, **21**, No. 9 (1960).

Note: Figure translations are in progress. See original paper for figures.