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Abstract

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ON THE ACOUSTIC RADIATION OF A TURBULENT FLOW IN THE PRESENCE OF ELASTIC BOUNDARIES

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The radiation of sound by a turbulent aerodynamic flow, when there are solid, rigid streamlined boundaries in the flow, was studied in ([1,2]). In doing so, the reasoning was based on the equation of sound generation by turbulence obtained in ([3]). In the present note an approximate calculation is set forth for acoustic radiation when aerodynamically thin streamlined elastic bodies are present in the flow. In contrast to ([1,2]), the investigation is carried out on the basis of the equation of sound propagation in a steady turbulent flow ([3,4])*. The statistical processes are assumed to be stationary, and equations for spectral amplitude densities are used throughout. The formalism developed below, however, can in some cases be generalized to statistical processes with slowly varying spatial and temporal averaged characteristics and, in particular, to locally stationary and homogeneous random fields.

Let us consider a certain bounded region Ω of a turbulent flow. We choose a moving coordinate system so that the mean flow velocity V in Ω is equal to zero. Following ([3]), we write the equation of sound propagation in a turbulent medium in the form

$$\Delta\rho - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\rho = -\frac{1}{c_0^2}\frac{\partial^2}{\partial x_i \partial x_j}T_{ij}, \tag{1}$$

where $T_{ij} = \rho v_i v_j + s_{ij} + (p - c_0^2\rho)\delta_{ij}$; ρ is the density; v_i are the components of the velocity fluctuations; s_{ij} is the tensor of viscous stresses; p is the pressure in the flow; c_0 is the speed of sound in a stationary medium. For $M = V/c_0 < 1$, one may approximately put $T_{ij} \approx \rho_0 v_i v_j + s_{ij}$, where ρ_0 is the density of an incompressible gas.

Passing to a fixed coordinate system and using the relation connecting the pressure fluctuation p^0 with the velocity fluctuation in a turbulent flow of an incompressible gas ([5]),

$$\frac{\partial^2}{\partial x_i \partial x_j} v_i v_j = -\frac{1}{\rho_0} \Delta p^0,$$

we write the equation for the spectral amplitude densities

$$\Delta \rho^{(1)}(\mathbf{r}_0) - \frac{1}{c_0^2} \left(-i\omega + V \frac{\partial}{\partial x_1} \right)^2 \rho^{(1)}(\mathbf{r}_0) = -\frac{1}{c_0^2} \frac{\partial^2}{\partial x_i \partial x_j} P_{ij}(\mathbf{r}_0), \tag{2}$$

where $P_{ij}(\mathbf{r}_0) \approx s_{ij}(\mathbf{r}_0) - p^0(\mathbf{r}_0)\delta_{ij}$.

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* Thus, the existence of a transverse shear of the mean velocity in the boundary layer near streamlined bodies and the sound refraction caused by this shear are not taken into account; only the "drift" of sound waves is considered. The presence of a transverse shear of the mean velocity in the turbulent boundary layer can be taken into account in calculating the fluctuations of pressure and viscous stresses in the flow, for example as was done in ([5]).

Suppose that inside Ω there are thin, i.e., strongly elongated in the direction of the flow, elastic bodies (for example, plates, rods, etc.), fixed relative to the chosen stationary coordinate system, and that sound propagation in the flow is governed by equation (2). We shall require that the following boundary conditions be satisfied:

$$\left. \frac{\partial \rho^{(1)}(\mathbf{r}_0)}{\partial \nu} \right|_s = -\frac{\rho_0}{c_0^2} \left(-i\omega + V \frac{\partial}{\partial x_1} \right)^2 w^{(1)}_{(\nu)}(\mathbf{r}_0), \tag{3}$$

$$\mathbf{t}(\mathbf{w}^{(1)})_{(\nu)} = -c_0^2 \rho^{(1)}(\mathbf{r}_0)|_s, \tag{4}$$

$$\mathbf{t}(\mathbf{w}^{(1)})_{(s)} = 0, \tag{5}$$

$$\mathbf{A}\mathbf{w}^{(1)}(\mathbf{r}_0) = -\frac{\partial}{\partial x_i} \left(c_{iklm} \varepsilon_{lm}(\mathbf{w}^{(1)}) \right) x_k^{(0)} = \beta \mathbf{w}^{(1)}(\mathbf{r}_0)|_{\Omega_1}. \tag{6}$$

In expressions (3)–(6), Ω_1 and s are, respectively, the volume and surface of the elastic bodies; $\mathbf{w}$ is the vector of elastic displacements; $\mathbf{t}(\mathbf{w}) = \tau_{ik} l_i x_k^{(0)}$ is the vector of stresses acting on an elementary surface area s; $\tau_{ik} = c_{iklm}\varepsilon_{lm}$ is the stress tensor; the subscript in parentheses (ν) or (s) denotes the projection of the corresponding vector onto the normal or onto the direction tangent to s; $\varepsilon_{lm}(\mathbf{w})$ is the strain tensor; $c_{iiii} = \lambda + 2\mu$; $c_{iikk} = \lambda$; $c_{ikik} = 2\mu$; $c'_{iklm} \equiv 0$; λ and μ are the Lamé coefficients; $\beta = -\omega^2/\rho_1$; ρ_1 is the density of the elastic body; $x_k^{(0)}$ is the unit vector of the x_k axis, and l_i is the direction cosine between $\vec{\nu}$ and x_i.

Introduce the auxiliary solution $\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0)$ of the adjoint problem, describing a regular (nonstatic) sound field and satisfying the equation

$$\Delta\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0) - \frac{1}{c_0^2}\left(-i\omega - V\frac{\partial}{\partial x_1}\right)^2 \tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0) = -\frac{1}{c_0^2}\delta(\mathbf{r}-\mathbf{r}_0), \tag{7}$$

adjoint to equation (2). We shall require that the solution $\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0)$ satisfy the boundary conditions adjoint to the boundary conditions (3)–(6). Multiply equation (2) by $\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0)$, and equation (7) by $-\rho^{(1)}(\mathbf{r}_0)$, add these equations, and integrate the right- and left-hand sides of the resulting expression over the volume Ω. To the volume integral on the left-hand side of the equality we apply Green' s theorem. As a result we obtain

$$-\int_{s+s_0}\left[\frac{\partial\rho^{(1)}(\mathbf{r}_0)}{\partial\nu}\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0) - \frac{\partial\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0)}{\partial\nu}\rho^{(1)}(\mathbf{r}_0)\right]ds(\mathbf{r}_0) -$$

$$-\frac{2i\omega}{c_0}M\int_{s_0}\rho^{(1)}(\mathbf{r}_0)\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0)\,ds(\mathbf{r}_0)\,l_1 +$$

$$+M^2\int_{s_0}\left[\frac{\partial\rho^{(1)}(\mathbf{r}_0)}{\partial\nu}\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0) - \frac{\partial\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0)}{\partial\nu}\rho^{(1)}(\mathbf{r}_0)\right]ds(\mathbf{r}_0)\,l_1 =$$

$$= \frac{1}{c_0^2}\rho^{(1)}(\mathbf{r}) - \frac{1}{c_0^2}\int_{\Omega}\frac{\partial^2}{\partial x_i\,\partial x_j}P^*_{ij}(\mathbf{r}_0)\tilde{\rho}^{(2)}(\mathbf{r}/\mathbf{r}_0)\,d\Omega(\mathbf{r}_0);$$

$$l_1 \equiv \cos(\widehat{\nu x_1}), \qquad M = V/c_0.$$

The integral over the surface s_0, which is the outer part of the surface bounding the volume Ω, is equal to zero by virtue of the radiation condition, since s_0 can be moved arbitrarily far away. The integral over the surface of the bodies s is equal to zero owing to the adjointness of the boundary conditions for equations (2) and (7) and to the self-adjointness of the operator of elasticity theory $\mathbf{A}$, which, as is known, follows from Betti' s theorem. Using the Ostrogradsky-Gauss theorem, we transform the volume integral on the right-hand side of the last expres-

and, taking into account the condition

$$\frac{\partial}{\partial x_j}P_{ij}(\mathbf{r}_0) = i\omega\rho_0 v_i(\mathbf{r}_0)\big|_s,$$

passing to sound pressures, we obtain

$$p^{(1)}(\mathbf{r}) = \int_{\Omega} P_{ij}(\mathbf{r}_0)\,\frac{\partial^2}{\partial x_i \partial x_j}\tilde{p}^{(2)}(\mathbf{r}/\mathbf{r}_0)\,d\Omega(\mathbf{r}_0)+$$

$$+\,i\omega\rho_0\int_s l_i v_i(\mathbf{r}_0)\,\tilde{p}^{(2)}(\mathbf{r}/\mathbf{r}_0)\,ds(\mathbf{r}_0) - \int_s l_i\frac{\partial}{\partial x_i}\tilde{p}^{(2)}(\mathbf{r}/\mathbf{r}_0)\,P_{ij}(\mathbf{r}_0)\,ds(\mathbf{r}_0). \quad (8)$$

It follows from solution (8) that the acoustic field of a turbulent flow in the presence of elastic bodies in the flow is a superposition of the radiation field of volume sources—the pulsations of pressures and viscous stresses in the flow —and of the fields of surface sources. The surface sources are the pulsations of pressure and viscous stresses and the velocity pulsations at the surface of the bodies, acting on the elastic bodies from the side of the flow. By the method of dimensional analysis, following (3), it may be established that the total radiation power of volume (quadrupole) sources is proportional to the ratio of the flow velocity to the speed of sound in the medium to the 8th power (M^8); the radiation power of surface (dipole) sources—the pulsations of pressure and viscous stresses—is proportional to M^6, and that of velocity pulsations (simple sources) to M^4. Consequently, in the case of subsonic turbulent flows ($M \ll 1$) and elastic but acoustically "soft" surfaces, when the velocity pulsations near the elastic surfaces in the flow are significant, the acoustic radiation of the flow will be due practically to the action of these pulsations. Conversely, if the bodies being flowed around are acoustically "rigid," the role of velocity pulsations is insignificant, and the radiation field is determined by the action of pressure and viscous-stress pulsations on the surface of the elastic bodies. In the cases noted, considerable sound radiation occurs from oscillating bodies—the so-called "pseudosound" is transformed into true sound.

On the basis of solution (8) one can calculate the mean-square value of the pressure in the radiation field of the flow. In particular, when velocity pulsations in the flow at the surface of elastic bodies may be neglected and $|p^0| \gg |s_{ij}|$, then, neglecting the field of volume sources, we obtain

$$\overline{|p^{(1)}(\mathbf{r})|^2} \simeq \int_s\int_s \frac{\partial}{\partial\nu}\tilde{p}^{(2)}(\mathbf{r}/\mathbf{r}_0')\frac{\partial}{\partial\nu}\tilde{p}^{(2)*}(\mathbf{r}/\mathbf{r}_0'')F(\mathbf{r}_0',\mathbf{r}_0'')\,ds(\mathbf{r}_0')\,ds(\mathbf{r}_0''), \quad (9)$$

where

$$F(\mathbf{r}_0',\mathbf{r}_0'') = \overline{p^0(\mathbf{r}_0')\,p^{0*}(\mathbf{r}_0'')}$$

is the correlation function of pressure pulsations at the surface of the body in the flow. In an analogous manner, the mean-square values of various characteristics of the acoustic field in other cases may be calculated.

Under the condition $M \ll 1$, the "acoustic wind" may be neglected, and in expressions (8) and (9) the auxiliary solution of the equation of sound propagation in a moving medium may be replaced by the solution of the equation for a stationary medium.

It follows from (9) that, if the auxiliary solution and the correlation function (in the present case, of pressure pulsations) are known, the solution of the problem of sound radiation by the flow reduces to the computation of quadratures (7).

In a number of cases, the mean values of products of field characteristics at two different points of space may be of interest; for example, the correlation of pressure amplitudes, etc. These quantities may be expressed in terms of the auxiliary diffraction fields, and for them formulas analogous to formula (9) may be obtained.

The radiation of sound by a flow in the presence of bodies in the flow has recently been studied by Doak ([8]) and Powell ([9]). The latter investigated the interaction of aerodynamic noise, radiated by a flow, with a plane rigid boundary. Doak confined himself to the consideration of absolutely soft and rigid bodies moving with the flow. The principal results of works ([8], 9) can be obtained on the basis of an analysis of the solution presented here.

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Note: Figure translations are in progress. See original paper for figures.