Physics

1961

SovietRxiv

Abstract

Full Text

Physics

B. N. Gershman

On Some Features of the Transverse Propagation of High-Frequency Waves in a Magnetoactive Plasma

(Presented by Academician M. A. Leontovich, October 7, 1960)

In the present paper we shall consider certain features of the propagation of high-frequency waves in a homogeneous plasma located in a constant magnetic field H_0. These features are associated with the presence of thermal motion of the electrons and are manifested sufficiently effectively only for propagation in a direction perpendicular to the field H_0 (transverse propagation), or in directions close to the indicated one. Below, the propagation is assumed to be strictly transverse.

For transverse propagation, using the method of the kinetic equation from nonrelativistic theory, it was concluded ([1−4]) that in frequency regions close to the electron gyrofrequency $\omega_{H,0}$, or to multiple frequencies $2\omega_{H,0}, 3\omega_{H,0}, \ldots$, there should appear narrow zones within which wave propagation is impossible. The assertion of the occurrence of such forbidden zones ("gaps") was first formulated by Gross. In his work ([1]), and later in ([2−4]), the gaps were investigated only for plasma waves (waves of type 3). The conclusion concerning the appearance of gaps can, in the nonrelativistic approach, also be extended to extraordinary and ordinary waves (waves 1 and 2).

The statements just made about gaps, however, require substantial modification. In particular, the results of ([1−4]) relating to this question must be reconsidered. The point is that the nonrelativistic analysis proves insufficient. For a correct solution it is necessary, in the first approximation, to which we shall restrict ourselves, to take into account the relativistic dependence of the electron mass on the velocity of its thermal motion.

Below, the simplest case of propagation of ordinary waves will be considered in comparative detail. The analogous investigation for waves 1 and 3 is more cumbersome, and here we shall restrict ourselves only to presenting the results obtained and to certain comparisons. In what follows we shall everywhere neglect the influence of collisions of electrons with other particles. The plasma is assumed not to be very strongly heated, so that the condition $\beta^2 = \varkappa T/m_0c^2 \ll 1$ is satisfied (m_0 is the rest mass of the electrons, T their temperature, $\varkappa$ Boltzmann's constant).

Proceeding to the analysis of the behavior of waves 2, we first present the results of the nonrelativistic calculation. In the case where the equilibrium distribution of electrons over velocities is Maxwellian, the dispersion equation is given in ([5]). From formula (35) of ([5]), under the condition

$$|\delta| \ll 1 \qquad \left(\delta = \frac{\varkappa T\tilde{k}^2}{m\omega_{H,0}^2} = \frac{\beta^2\tilde{n}^2}{u}\right) \tag{1}$$

we arrive at the approximate equation

$$\tilde{n}_2^2 = 1 - v\left\{1 + \frac{\delta}{1-u} + \frac{\delta^2}{4(1-4u)} + \frac{\delta^3}{24(1-9u)}\right\}. \tag{2}$$

Condition (1) is usually violated only in the case of weak anisotropy. In relations (1) and (2) the following notation is used: $\omega_{H,0} = eH_0/m_0c$ is the electron gyrofrequency (e is the absolute value of the electron charge); $v = 4\pi e^2N/m\omega^2$ (N is the electron concentration); $u = \omega_{H,0}^2/\omega^2$; $\tilde{k} = k - iq$ is the complex wave number; $\tilde{n} = n - i\chi$ is the complex refractive index (q is the amplitude absorption coefficient, χ is the absorption index). Equation (2) was obtained by using an expansion in powers of the small parameter δ (terms $\sim \delta^4$ were neglected).

Without taking into account the thermal motion of the electrons, $\delta = 0$. Then from (2) we arrive at the simple formula $\tilde{n}_2^2 = 1 - v$, according to which propagation is possible for $v < 1$. In the opposite case $\tilde{n}_2^2 < 0$, and propagation is forbidden (for $\tilde{n}_2^2 < 0$ in a homogeneous medium the fields, when varied along the z-axis, decrease without oscillating, according to the law $\exp\left(-\frac{\omega}{c}|\tilde{n}_2|z\right)$).

If, however, $\delta \neq 0$, then from equation (2) it is easy to establish that for $v < 1$ propagation is no longer possible for all frequencies ω. In the regions where $\omega \simeq \omega_{H,0}, 2\omega_{H,0}, 3\omega_{H,0}$ (or, respectively, $u \simeq 1, 1/4, 1/9$) there arise narrow bands of width $\Delta\omega$ in which $\tilde{n}_2^2 < 0$. For the principal purpose, in the region $\omega \simeq \omega_{H,0}$, from (2) we obtain $\Delta\omega/\omega = \beta^2 v/2 \ll 1$. For the resonances $\omega \simeq 2\omega_{H,0}$ and $\omega \simeq 3\omega_{H,0}$ we have, respectively, the estimates $\Delta\omega/\omega \lesssim \beta^4 v$ and $\Delta\omega/\omega \lesssim \beta^6 v$. Thus, for wave 2, just as for wave 3 ([1–4]), in the nonrelativistic approximation one may pose the question of the occurrence of gaps, whose relative width decreases as the number of the gyroresonance increases. As has already been indicated, for a final solution of the question of gaps one must take relativistic corrections into account. Some questions of the relativistic kinetic theory of wave propagation in a magnetoactive plasma were considered in ([6,7]). Here, however, there is no need to rely on these works, since, in fact, with some changes one may use the nonrelativistic dispersion equations ([5]). In these equations it is necessary only, before carrying out the averaging over velocities **V**, to replace, in the terms containing gyroresonance terms, the gyrofrequency

$\omega_{H,0}$ by

$$\omega_H = \omega_{H,0}\sqrt{1-\frac{V^2}{c^2}} \simeq \omega_{H,0}\left(1-\frac{V^2}{2c^2}\right).$$

Then, for transverse propagation of wave 2 under condition (1), from formulas (5), (10), (14) and the first of relations (15) of work ([5]), we arrive at the equation

$$-\frac{1}{\sqrt{2\pi}}\left(\frac{m_0}{\chi T}\right)^{5/2} v\int_0^{\infty}\int_{-\infty}^{+\infty}\left\{1+\frac{\tilde{n}^2V_\rho^2}{2uc^2}\left[\frac{1}{1-u\left(1-V^2/c^2\right)}\right]+\frac{\tilde{n}^4V_\rho^4}{32u^2c^4}\left[\frac{1}{1-4u\left(1-V^2/c^2\right)}\right]+\frac{\tilde{n}^6V_\rho^6}{1152u^3c^6}\left[\frac{}{1\cdot}\right.\right. \tag{3}$$

The transition to the nonrelativistic relation (2) can formally be carried out by assuming in the denominators of the expressions in square brackets appearing in (3) that $V^2/c^2 = 0$. However, such neglect of terms $\sim V^2/c^2$ is not legitimate in the immediate vicinity of the frequencies $\omega_{H,0}, 2\omega_{H,0}, 3\omega_{H,0}$. The necessity of taking relativistic effects into account when solving the problem of gaps is connected precisely with this circumstance.

Let us now consider, on the basis of equation (3), the behavior of wave 2 in the region of the principal gyroresonance, i.e., for $\omega \simeq \omega_{H,0}$. Keeping in (3) the first two terms in the curly brackets, when integrating over velocities we pass from the variables V_ρ and V_z, which have the meaning of the components of the velocity $\mathbf{V}$ in a cylindrical coordinate system, to the variables V and θ ($V_\rho = V\sin\theta;\ V_z = V\cos\theta$). Integrating over the angle θ and carrying out the simple-

transformations, we obtain

$$1-v = \tilde{n}^2\left\{1+\frac{v}{15}\left[3+\frac{u-1}{\beta^2}+\frac{(u-1)^2}{\beta^4}-\frac{(\sqrt{u}-1)^5}{\beta^5}\sqrt{\frac{1}{8}}\left(Y(z)-Y(-z)\right)\right]\right\}, \tag{4}$$

where $z = c\sqrt{(u-1)m_0/2\varkappa T}$, and for $Y(z)$ we have

$$Y(z) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{e^{-x^2}}{z-x}\,dx. \tag{5}$$

It is convenient to use the equation in the form (4) in the case when

$$\omega < \omega_{H,0} \quad (u > 1). \tag{6}$$

Under the restriction (6), the integrand in (5) has a singularity on the real axis. For this reason the integration must be carried out along a contour running from $-\infty$ to $+\infty$ with the point $x = z$ bypassed. In the present case, in establishing the bypass rule one may use the restriction according to which, in a plasma with an equilibrium Maxwellian distribution, the possibility of growth of weak perturbations is excluded. Taking this circumstance into account, one can obtain $Y(\pm z) = R(\pm z) \pm \pi i e^{-z^2}$, where $R(z)$ is real. The presence of an imaginary part $\sim e^{-z^2}$ entails the conclusion that there appears a specific absorption not associated with collisions.

For $z \gg 1$, using the expansion $R(z) \simeq \frac{1}{z}\left(1 + \frac{2}{z} + \frac{4}{3z^3} + \frac{8}{15z^5} + \cdots\right)$ and neglecting the small absorption, we arrive at the formula

$$\tilde{n}_2^2 = \frac{1-v}{1+\beta^2 v/(1-u)}. \tag{7}$$

Formula (7) coincides with the nonrelativistic relation that follows from equation (2) if in the latter the terms with δ^2 and δ^3 are omitted. Thus, the relativistic corrections are small under the condition $|1-u| \gg \beta^2$. Earlier, in determining the width of the "gap," we used equation (2) also for $|1-u| \lesssim \beta^2$, but this operation, as is clear from what follows, was illegitimate.

For $z \ll 1$, or, in another notation, when

$$|1-u| \ll \beta^2 \tag{8}$$

the specific absorption is also small (see below (10)). Neglecting it and using the expansion $R(z) \simeq z\left(1 - \frac{2}{3}z^2 + \cdots\right)$, we have

$$\tilde{n}_2^2 = \frac{1-v}{1+v/5}. \tag{9}$$

For $v < 1$ and under the restrictions (6), (8), it followed from the nonrelativistic equation (3) that $\tilde{n}_2^2 < 0$. This corresponded to the occurrence of a gap. However, in fact, under the same restrictions, according to (9), $\tilde{n}_2^2 > 0$, and no gap appears (we note that $\tilde{n}_2^2 > 0$ also in the region $|1-u| \gtrsim \beta^2$, if the specific absorption is not taken into account). Thus, the assertion of the appearance for wave 2 of narrow forbidden zones turns out to be incorrect even for the fundamental resonance, to say nothing of higher resonances (see below).

The main feature of the behavior of waves 2 in the region $\omega \simeq \omega_{H,0}$ is connected with the conclusion that there is appreciable specific absorption. The latter is most significant for $|1-u| \sim \beta^2$. For an approximate determination

of this absorption (with an error of no more than 20%) one may use the formula

$$\frac{q_2}{n_2} = \frac{\chi_2}{n_2} \simeq \frac{2\sqrt{\pi}\, v}{15}\, z^{5/2} e^{-z^2}. \tag{10}$$

For $z \ll 1$, $\chi/n \sim z^{5/2} \ll 1$. The value of the ratio χ/n in (10) is maximal at $z = \sqrt{5/4}$. In this case $\chi_2/n_2 \simeq 0.2v$. Since propagation of wave 2 is possible only for $v < 1$, always $\chi_2 < n_2$, or even $\chi_2 \ll n_2$. Nevertheless, the maximum values of the absorption coefficient χ_2 are quite substantial.

Under the condition $\omega > \omega_{H,0}$, the reverse of (6), it is more convenient to deal not with integrals of the form (5), but to obtain an equation similar to (4), containing the probability integral. Under the restrictions $1 - u \gg \beta^2$ or $1 - u \ll \beta^2$ (8), we again arrive at formulas (7) and (9). However, in contrast to the case (6), the specific absorption here is altogether absent.

Using equation (3), one can analyze the behavior of wave 2 in the frequency regions $\omega \simeq 2\omega_{H,0}$, $3\omega_{H,0}$. For these resonances we again come to the conclusion that the gaps are fictitious. Moreover, in the relativistic approach it is not possible to establish any substantial features. Thus, for $\omega \simeq 2\omega_{H,0}$, the possible changes in the values of the refractive index Δn (relative to the value $n_2 = \sqrt{1-v}$) are such that $|\Delta n_2/n_2| \lesssim \beta^2 \ll 1$. For the specific absorption at $\omega \simeq 3\omega_{H,0}$ we have the restriction $\chi_2/n_2 \lesssim \beta^2$. For the resonance at $\omega \simeq 3\omega_{H,0}$, the estimates $|\Delta n_2/n_2| \lesssim \beta^4$ and $\chi_2/n_2 \lesssim \beta^4$ are valid.

We now present several conclusions concerning the transverse propagation of waves 1 and 3. For the extraordinary wave 1 the principal resonance is at $\omega \simeq 2\omega_{H,0}$, and for the plasma wave 3—at $\omega \simeq 3\omega_{H,0}$. In these resonance regions $|\Delta n/n| \lesssim 1$ and $\chi/n \lesssim 1$. At $\omega \simeq \omega_{H,0}$, for wave 1 no noticeable changes in the character of propagation occur, and the specific absorption is small, so that $\chi_1/n_1 \lesssim \beta^2$. An analogous situation for this wave also arises at $\omega \simeq 3\omega_{H,0}$. For wave 3, propagation is impossible in a broad frequency region that includes the interval near $\omega \simeq \omega_{H,0}$ and borders on the region where $\omega \simeq 2\omega_{H,0}$ [8]. In a narrow frequency range, noticeable features arise only at $\omega \simeq 3\omega_{H,0}$. But these features in no way correspond to the anomalies considered in [1-4]. For the resonances $\omega \simeq 4\omega_{H,0}, 5\omega_{H,0}, ...$, contrary to the conclusions of [1-4], no substantial irregularities in the behavior of the plasma waves arise when condition (1) is well satisfied. Naturally, there is no question here of the appearance of gaps.

Research Radiophysics Institute
at the Gorky State University
named after N. I. Lobachevsky

Received
8 IX 1960

REFERENCES

1. E. P. Gross, Phys. Rev., **82**, 232 (1951).
2. A. G. Sitenko, K. N. Stepanov, ZhETF, **31**, 642 (1956).

3. K. N. Stepanov, V. S. Tkalich, ZhETF, **28**, 1789 (1958).

4. J. Bernstein, Phys. Rev., **109**, 10 (1958).

5. B. N. Gershman, ZhETF, **24**, 659 (1953).

6. B. A. Trubnikov, *Plasma Physics and Problems of Controlled Thermonuclear Reactions*, **3**, USSR Academy of Sciences Press, 1958, p. 104.

7. B. N. Gershman, *Izv. Vyssh. uchebn. zaved., Radiofizika*, **3**, 534 (1960).

8. B. N. Gershman, ZhETF, **38**, 912 (1960).

Note: Figure translations are in progress. See original paper for figures.