PHYSICS

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1961

SovietRxiv

Abstract

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ON THE PHENOMENON OF "SCATTERING" OF CHARGED QUASIPARTICLES AT SINGULAR POINTS IN p-SPACE

All the kinetic and resonance properties of metals and semiconductors are connected with the dynamics of the motion of quasiparticles, which are the carriers of conductivity in these bodies. In the ideal-gas approximation, sufficient for the study of most effects, the dynamics of the motion is determined by the complex dispersion law $\varepsilon = \varepsilon(\mathbf{p})$, an important characteristic of which is the geometry of the isoenergetic surface. The equations of motion in an external electromagnetic field in this case are the usual Lorentz equations:

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E} + \frac{e}{c}[\mathbf{v}\mathbf{H}], \qquad \mathbf{v} = \nabla_p \varepsilon.$$

Here $\mathbf{v}$ is the particle velocity, a complex function of the quasimomentum $\mathbf{p}$.

In the present note we shall consider a peculiar phenomenon of "scattering" of quasiparticles, associated with singularities of the dispersion law, and not with the presence of a scattering force center.

As is known, in motion in a constant magnetic field the particle energy ε and the projection of the momentum p_H on the direction of the magnetic field are conserved. If the trajectory $\varepsilon = \text{const}$, $p_H = \text{const}$ is closed, then the particle performs periodic motion along it with period $T = m^* c/eH$, where $m^* = m^*(\varepsilon, p_H)$ is the effective mass of the particle for the given section (^{1}). If the magnetic field varies slowly in space and time (i.e. $\alpha_H = \dot{H}T/H \ll 1$) or in the presence of a weak longitudinal electric field ($\alpha_E = E/H \ll 1$), the parameters p_H and ε are no longer constant. However, in this case p_H and ε change so slowly that the picture of motion in p-space may be represented as drift, rotation, and deformation of the "current lobe," i.e. of the contour $\varepsilon = \text{const}$, $p_H = \text{const}$, along which the particle rotates. The drift velocity is determined by the rate of change of the parameters p_H and ε. The motion of the particle in r-space may be represented as rapid oscillations about the "center of the orbit" $\mathbf{R}$ and a smooth displacement of this center in the direction of the magnetic field. The velocity of the center of the orbit $\dot{\mathbf{R}}$ is given by the obvious equation $\dot{\mathbf{R}} = \bar{v}_H \vec{\xi}$, where $\vec{\xi} = \mathbf{H}/H,\ \mathbf{H} = \mathbf{H}(\mathbf{R}, t)$, while the averaged velocity $\bar{v}_H$ is determined by the formula

$$v_H(P_H, \bar{\varepsilon}, \vec{\xi}) = -\frac{1}{2\pi m^*}\frac{\partial S}{\partial P_H}(P_H, \bar{\varepsilon}, \vec{\xi})$$

$\left(S\right.$ is the area of the section $\varepsilon = \bar{\varepsilon},\ p_H = P_H\big)$. The averaged quantities $P_H = \bar{p}_H$, $\bar{\varepsilon}$, and the unit vector of the magnetic field $\vec{\xi}$ specify the position of the "current lobe" in momentum space. An important characteristic of the averaged motion in fields of the type indicated above is the adiabatic

the invariance of the quantity $S(P_H, \bar{\varepsilon}, \vec{\xi})/H(\mathbf{R}, t)$, which makes it possible partially to integrate the averaged differential equations. For a complete specification of the motion it is necessary to know an additional equation for one of the quantities P_H or $\bar{\varepsilon}$.

We shall give these equations for three types of fields:

1. In motion in a weakly inhomogeneous constant magnetic field, this equation is $\bar{\varepsilon} = \text{const}$.
2. For the case of parallel homogeneous electric and magnetic fields we have: $\dot{P}_H = eE$.
3. If a time-dependent magnetic field $\mathbf{H}(t)$ acts on the particle, it is necessary to take into account the electric field that arises in this case, which leads to nonconservation of $\bar{\varepsilon}$. In this case the equation for P_H has the simplest form:

$$\dot{P}_H = \frac{1}{m^*}\frac{\partial}{\partial \bar{\varepsilon}}\oint \mathbf{P}_\perp\, dS,$$

where $\mathbf{P}_\perp$ is the projection of the momentum onto the plane perpendicular to $\vec{\xi}$.

The equations obtained (together with the equation $S/H = \text{const}$) will be used by us in the study of the drift of a "current petal" through saddle points of an isoenergetic surface (it is assumed that the surface is not everywhere convex). A saddle point is special with respect to the dynamics of motion in a homogeneous magnetic field, when the plane of the "current petal"is tangent to the isoenergetic surface at this point. In the present case the contour along which the motion in the magnetic field occurs has the form of a curve with self-intersection (a "figure eight" in Fig. 1). The special point of the "figure eight" (the point of self-intersection) coincides with the saddle point of the isoenergetic surface. The projection of the particle velocity $v_\perp$ onto the plane of the contour goes to zero at the special point: $v_\perp \sim |\mathbf{p} - \mathbf{b}|$ ($\mathbf{b}$ is a vector in $\mathbf{p}$-space drawn to the special point). This means that the period of revolution of the particle in the magnetic field diverges logarithmically as $\rho_H \to b_H$.

Fig. 1

Figure 1: Fig. 1

Fig. 1

If the plane of a "current petal" drifting in $\mathbf{p}$-space intersects a saddle point of the isoenergetic surface, then the "current petal" splits into two "current petals," whose contours are determined by regions I and II (Fig. 1), separated by the special point. The character of the motion in each of the regions is completely different. On the other hand, the exact (microscopic) initial conditions that determine whether a particle enters region I or II become mixed very rapidly, so that in each macroscopic element of the isoenergetic surface there are points from which the particle enters both the first and the second region. From this point of view one may regard the entry of particles into each of the regions as a random process and speak of a kind of "scattering" of particles near the special point; here the scattering probabilities into I and II (w_1 and w_2) have quite definite values, which will be determined below. It should be especially emphasized that the peculiarity of such a scattering process is connected with the absence of any force scattering center in $\mathbf{r}$-space; the scattering event itself may take place at any point of $\mathbf{r}$-space.

Consider the difference $\Delta(t) = \rho_H(t) - b_H(t)$, where $b_H(t)$ corresponds

saddle point on the surface $\varepsilon = \varepsilon(t)$, the normal vector to which is parallel to $\vec{\xi}(t) = \mathbf{H}/H$. On the last turn before entering region I or II, each particle crosses the principal line of curvature ($p_2 = 0$ in Fig. 1) on the isoenergetic surface $\varepsilon = \varepsilon(0)$ at a certain value of the difference $\Delta(0) = p_H(0) - b_H(0)$. Having made a complete circuit around one of the loops of the "figure eight," the particle again finds itself in the vicinity of the self-intersection point $b(t)$. Depending on the sign of the difference $\Delta(T)$ at this instant of time, the particle enters region I or II, while the quantity $\Delta(T)$ is uniquely determined by the value of $\Delta(0)$ at the beginning of the circuit. From what has been said it is clear that the difference $\Delta(0)$ is naturally to be taken as the impact parameter characterizing a particle in the scattering ensemble. To regions I and II there correspond elements δ_I and δ_{II} of the values $\Delta p_H = \Delta(0)$ that determine entry into these regions. The scattering probabilities (w_1 and w_2), i.e., the relative numbers of particles entering, respectively, regions I and II, are proportional to the fluxes in $\mathbf{p}$-space through the elements δ_I and δ_{II}. If the distribution function is assumed sufficiently smooth, then, to first order in the small parameter α, these fluxes are proportional to the magnitudes of the elements themselves.

From the equations of motion, for small α we have $dt = \dfrac{c}{eH}\dfrac{dp_l}{v_\perp}$, where dp_l is the differential of the arc length of the "figure eight." For particles that have made a complete circuit around one of the loops of the "figure eight," we obtain

$$\Delta^I(T_I) - \Delta^I(0) = \frac{c}{eH}\oint_I (\dot{p}_H - \dot{b}_H)\frac{dp_l}{v_\perp}, \tag{1}$$

and, correspondingly, for $\Delta^{II}(T_{II})$ (the integration is carried out along the contour of the corresponding loop of the "figure eight"). According to the above, the sought elements δ_I and δ_{II} must be determined from the conditions $\Delta^{I,II}(T_{I,II}) = 0$. This gives

$$\frac{\delta_I}{\delta_{II}} = \oint_I (\dot{p}_H - \dot{b}_H)\frac{dp_l}{v_\perp}\Big/ \oint_{II} (\dot{p}_H - \dot{b}_H)\frac{dp_l}{v_\perp}. \tag{2}$$

The concrete evaluation of these contour integrals depends on the character of the external field in which the particles move. We shall give results for several cases (using the averaged equations of motion).

1. **A weakly inhomogeneous field constant in time.** The formula for the ratio of the probabilities has the form

$$\frac{w_1}{w_2} = \frac{\partial}{\partial l}\left(\frac{S_1}{H}\right)\Big/\frac{\partial}{\partial l}\left(\frac{S_2}{H}\right), \tag{3}$$

where l is the length along a magnetic-field line; $S_{1,2}(\varepsilon, b_H(\vec{\xi}), \vec{\xi})$ $(\vec{\xi} = \vec{\xi}(l))$ are the areas of the sections of each of the loops in the section of the surface $\varepsilon = \text{const}$ by a plane passing through the singular point and perpendicular to $\vec{\xi}(l)$. For a straight field line formula (3) simplifies: $w_1/w_2 = S_1/S_2$.

2. **Parallel electric and magnetic fields, constant in time.** For the ratio of the probabilities in this case we obtain

$$\frac{w_1}{w_2} = \frac{d}{dp_H}S_1(p_H, \varepsilon_{\text{cr}}(p_H))\Big/\frac{d}{dp_H}S_2(p_H, \varepsilon_{\text{cr}}(p_H)),$$

where $\varepsilon_{\text{cr}}(p_H)$ determines that isoenergetic surface $\varepsilon = \varepsilon_{\text{cr}}(p_H)$ on which, for a given value of p_H, the singular point lies.

It should be noted that the drift time through the singular point, i.e., the "scattering time," turns out to be $\sim T\ln\alpha$; therefore, for the possibility of realizing

for the effect it is necessary that the inequality $\tau \gg T\ln\alpha$ be satisfied (τ is the mean free time). This inequality is well satisfied for $H \sim 10^4$ oersted and a mean free path $\gtrsim 0.1$ cm, and imposes practically no restrictions on α. On the other hand, all calculations were carried out in the classical approximation, i.e., the possibility of tunneling transitions from trajectories of one region to another was not taken into account. The latter is valid under the condition $v_H\delta p_H \gg \hbar/T$, where δp_H is the change of p_H in one turn; for excessively small field inhomogeneities a quantum treatment of the scattering problem is necessary.

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Received 2 I 1961

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Note: Figure translations are in progress. See original paper for figures.