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1961

SovietRxiv

Abstract

Full Text

HYDROMECHANICS

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MOTION WITH PLANE WAVES OF AN INCOMPRESSIBLE CONDUCTING FLUID WITH ELECTROMAGNETIC RADIATION TAKEN INTO ACCOUNT

(Presented by Academician L. I. Sedov, 12 X 1960)

In the present work we consider the problem of oscillations of a plane layer of finite width of an electrically conducting incompressible fluid. These oscillations arise owing to the release in the fluid, at the initial instant of time, of energy E and of a counterpressure p on the free surface, produced by an external magnetic field and the external medium. During the oscillatory motion an electromagnetic wave is emitted from the free surface. This wave carries away to infinity the energy of the moving fluid. Thus the oscillations will be damped, and at a certain instant of time the motion will cease.

Let a plane element of width $2a$ of conducting fluid be given, whose electrical conductivity we shall henceforth assume to be infinite ($\sigma = \infty$). We direct the coordinate axes as indicated in Fig. 1. The dashed lines in the figure denote the position of the fluid at the initial instant of time $t = 0$. Suppose that through the fluid there flows a distributed current I, parallel to the y-axis. The density of this current j will henceforth be assumed constant. The distribution of the magnetic field H_z at the initial instant of time outside and inside the fluid is uniquely determined by this current.

When, at the initial instant of time $t = 0$, energy with density E_0 is released in the yz-plane, the entire fluid is torn into two parts, which will move in different directions along the x-axis. The solid lines in Fig. 1 show the position of the fluid at the time $t = t_1$. We shall further assume that in the space between the separating parts of the fluid there is no pressure. The presence of an external counterpressure, determined by the magnetic field H_z and the external nonconducting medium, leads to the fact that at the time $t = t_2$, determined by the solution of the problem, the fluid will stop and begin to move in the opposite direction. The presence of electromagnetic radiation accelerates this stopping. Each of the parts of the fluid returns to its initial position, having a nonzero velocity v_1. We shall assume the collision of the parts of the fluid to be absolutely elastic, so that after the collision each of the parts of the fluid will have the initial velocity equal to v_1, and so on.

We take the electric field at the initial instant of time $t = 0$ to be zero.

We solve the problem in Lagrangian variables. Denote by x_0 the position of a fluid particle at the time $t = 0$, and by x its position at the time t. From the continuity equation we obtain:

$$x = x_0 + A(t). \tag{1}$$

The distribution of the magnetic field among the particles, by virtue of infinite conductivity, will remain unchanged, i.e., will coincide with the distribution at the time $t = 0$.

At $t = 0$ we have:

$$H_z = -\frac{I}{2c}\frac{x_0}{a}, \qquad |x_0| \leqslant a;$$

$$H_z = -\frac{I}{2c}\operatorname{sign} x_0, \qquad |x_0| \geqslant a.$$

Thus, in the moving fluid,

$$H_z = -\frac{I}{2c}\frac{x_0}{a}. \tag{2}$$

To find the pressure distribution $p(x_0, t)$ inside the fluid, we use the equation of motion:

$$A''(t) = -\frac{1}{\rho}\frac{\partial}{\partial x_0}\left(p + \frac{H_z^2}{8\pi}\right). \tag{3}$$

We integrate equation (3) with respect to x_0. We obtain

$$x_0 A''(t) = -\frac{1}{\rho}\left(p + \frac{H_z^2}{8\pi}\right) + B(t). \tag{4}$$

In equation (4), $B(t)$ is, for the time being, an arbitrary function. The functions $A(t)$ and $B(t)$ are determined from the boundary conditions. At the boundaries of the fluid the total pressure $P = p + H_z^2/8\pi$ is continuous; consequently:

$$\text{for } x_0 = 0: \qquad p + \frac{H_z^2}{8\pi} = 0; \tag{5}$$

$$\text{for } x_0 = a: \qquad p + \frac{H_z^2}{8\pi} = p_1 + \frac{H_{z1}^2}{8\pi}. \tag{6}$$

Fig. 1

Figure 1: Fig. 1

From equation (4) and condition (5) we find that $B(t) = 0$. In the boundary condition (6), p_1 is the external hydrodynamic pressure, and H_{z1} is the intensity of the external magnetic field at $x_0 = a$, which does not coincide with the field intensity determined by formula (2). The presence of a jump in the magnetic field is explained by the occurrence, on the free surface of the fluid $x_0 = a$, of a surface current $i(t)$.

Fig. 1

To determine $H_{z1}(t, a)$, we solve the external problem for the electric and magnetic fields:

$$\frac{\partial^2 E_y}{\partial t^2} = c^2 \frac{\partial^2 E_y}{\partial x^2}, \tag{7}$$

$$\frac{\partial H_{z1}}{\partial t} = c \frac{\partial E_y}{\partial x}. \tag{8}$$

Under the initial and boundary conditions

$$t = 0: \qquad E_y = 0, \quad H_{z1} = -\frac{I}{2c}; \tag{9}$$

$$x_0 = a: \qquad E_y = -\frac{A'(t)}{c} H_{z1}. \tag{10}$$

The last condition follows from the continuity of E_y in the coordinate system associated with the moving boundary $x_0 = 0$.

We take the solution of the equation for E_y in the form of an outgoing wave. From equation (8) and conditions (9) and (10) we find H_{z1} at $x_0 = a$:

$$H_{z1}(t, a) = -\frac{I}{2c} \frac{1}{1 - A'(t)/c}. \tag{11}$$

Substituting $H_{z1}(t, a)$ from (11) into equation (6) and neglecting terms of order higher than $A'(t)/c$, we obtain

$$p(t, a) = p_1 + \frac{I^2}{2c^2} \frac{A'(t)}{8\pi c}. \tag{12}$$

Thus, for $x_0 = a$, it follows from equation (4) that

$$A''(t) + \frac{I^2}{16\pi\rho a c^2}\frac{A'(t)}{c} = -\frac{1}{a\rho}\left(p_1 + \frac{I^2}{32\pi c^2}\right). \tag{13}$$

The initial conditions for the equation obtained are found from relation (1) and the energy balance at the initial instant:

$$\text{for } t = 0: \qquad A(0) = 0, \qquad A'(0) = \sqrt{\frac{E_0}{\rho a}}. \tag{14}$$

We transform equation (13) to dimensionless quantities

$$\tau = \sqrt{\frac{p_1 + I^2/32\pi c^2}{\rho}}\,\frac{t}{a}, \qquad u = \frac{A(t)}{a},$$

$$\mathcal{E} = \frac{E_0}{\left(p_1 + \dfrac{I^2}{32\pi c^2}\right)a}, \qquad \alpha = \frac{I^2}{16\pi c^3\sqrt{\rho\,(p_1 + I^2/32\pi c^2)}}.$$

Substitution of the variables in (13) and (14) gives

$$\frac{d^2u}{d\tau^2} + \alpha\frac{du}{d\tau} = -1; \tag{15}$$

$$u(0) = 0, \qquad \left(\frac{du}{d\tau}\right)_{\tau=0} = \sqrt{\mathcal{E}} = v_0. \tag{16}$$

The quantity $\alpha\, du/d\tau$ in equation (15) is called, in the theory of oscillations, a friction force. In the present problem the presence of this force is connected with electromagnetic radiation and leads to damped oscillations.

The solution of equation (15) under the initial conditions (16) is

$$u(\tau) = \frac{\alpha v_0 + 1}{\alpha^2}\left(1 - e^{-\alpha\tau}\right) - \frac{\tau}{\alpha}. \tag{17}$$

Solving the equation $u(\tau) = 0$ gives the value of the time $\tau = \tau_1$ at which the moving liquid returns to its initial position. The quantity $-(du/d\tau)_{\tau=\tau_1}$ then gives the initial velocity of the next cycle of motion, and so on. Determination of τ_1 reduces to solving a transcendental equation. For a qualitative investigation of solution (17), let us find τ_1 accurate to terms of order α:

$$\tau_1 = 2v_0 - \frac{\alpha}{6}v_0^2. \tag{18}$$

Substituting τ_1 into the expression $-du/d\tau$, we find the initial velocity of the second cycle of oscillations

$$v_1 = v_0 - \frac{\alpha}{6} v_0^2. \tag{19}$$

For the initial velocity of the $(k+1)$-st cycle of oscillations we thus obtain

$$v_k = v_0 - \frac{\alpha k}{6} v_0^2. \tag{20}$$

The amplitude of the oscillations is found from the energy integral of equation (16)

$$u_{k\,\max} = \frac{v_k}{\alpha} + \frac{1}{\alpha^2} \ln \frac{1}{1 + \alpha v_k}. \tag{21}$$

From equation (20) we find k for which the motion ceases in the first approximation. Setting $v_k = 0$ in equation (20), we obtain $k = 6/\alpha v_0$.

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Received
10 X 1960

Note: Figure translations are in progress. See original paper for figures.

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