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PHYSICS

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Abstract

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PHYSICS

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DISTRIBUTION OF HETEROCHARGE AND FIELD IN PHOTOELECTRETS

(Presented by Academician A. V. Shubnikov, 9 XII 1960)

1. Let us consider a dielectric for which the band diagram and the scheme of electronic transitions are shown in Fig. 1. Such a model corresponds to modern ideas about crystals forming photoelectrets ([1,2]).

The stationary state of a photoelectret in the polarizing field E_1 is described in dimensionless notation* by the following system of equations and boundary conditions:

$$dn/dx + nE = 0; \qquad N = \frac{M_2 n}{b+n};$$

$$dE/dx - \varrho = 0; \qquad P = \frac{aM_1}{a+n}; \tag{1}$$

$$\varrho = P - N - n;$$

$$E_0 = E_1/x; \qquad E_l = E_1/x. \tag{2}$$

The solution of system (1) has the following form ([3]):

$$n = n_0 e^{\psi};$$

$$E = \left[E_0^2 + 2M_1 \ln \frac{n_0 + ae^{-\psi}}{n_0 + a} + 2M_2 \ln \frac{b + n_0 e^{\psi}}{b + n_0} - 2n_0(1 - e^{\psi})\right]^{1/2}; \tag{3}$$

$$x = \int_{\psi}^{0} \left[E_0^2 + 2M_1 \ln \frac{n_0 + ae^{-\psi}}{n_0 + a} + 2M_2 \ln \frac{b + n_0 e^{\psi}}{b + n_0} - 2n_0(1 - e^{\psi})\right]^{-1/2} d\psi.$$

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The potential difference established across the photoelectret in the polarizing field E_1 is found from the last equation (3) as $V = -\psi(l)$. The near-anode concentration of conduction electrons n_0 is determined by the equation

$$\left[\frac{\left(1+\frac{a}{n_0}e^V\right)}{\left(1+\frac{a}{n_0}\right)}\right]^{M_1/M_2}\frac{\left(1+\frac{b}{n_0}e^V\right)}{\left(1+\frac{b}{n_0}\right)} = e^{V+\frac{n_0}{M_2}(1-e^{-V})}, \tag{4}$$

which follows from (2) and expresses the condition of absence of homocharge

$$\left(\int_0^l \varrho(x)\,dx = 0\right).$$

2. On the basis of experimental data and estimates, one may assume that in most cases there is weak excitation of donors

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* The units of measurement are: ν; $\bar{x} = [\varkappa kT/4\pi q^2\nu]^{1/2}$, $\bar{\psi} = kT/q$; $\bar{E} = kT/qx$; $\bar{\rho} = q\nu$. The unit of concentration ν may be chosen arbitrarily. The parameters $a = k_1^*/\alpha^*\nu$; $b = k_2^*/\beta^*\nu$. Dimensional quantities are marked with an asterisk.

$(P \ll M_1;\quad k_1^*M_1^* = a_1)$ (2) and weak filling of the acceptors $(N \ll M_2)$ (4) in photoelectrets. In this case $a'/n \ll 1,\ b/n \gg 1$, and the solution has the form

$$n = n_1 e^{\frac{V}{2}+\psi}; \qquad N = \frac{M_2 n_1}{b}e^{-\frac{V}{2}+\psi}; \qquad P = \frac{aM_1}{n_1}e^{-\frac{V}{2}-\psi};$$

$$\rho = -\frac{2}{L^2}\,\mathrm{sh}\left(\frac{V}{2}+\psi\right); \qquad E = \left\{E_0^2 + \frac{4}{L^2}\left[\mathrm{ch}\left(\frac{V}{2}+\psi\right) - \mathrm{ch}\,\frac{V}{2}\right]\right\}^{1/2}, \tag{5}$$

$$x = \int_\psi^0 \left\{E_0^2 + \frac{4}{L^2}\left[\mathrm{ch}\left(\frac{V}{2}+\psi\right) - \mathrm{ch}\,\frac{V}{2}\right]\right\}^{-1/2} d\psi.$$

The parameters

$$n_1 = \left[\frac{aM_1 b}{M_2+b}\right]^{1/2}; \qquad L = \left[\frac{b}{aM_1(M_2+b)}\right]^{1/4} = (n_1+N_1)^{-1/2} \tag{6}$$

characterize the properties of the crystal under illumination in the absence of a polarizing field (i.e., in the de-electret state). They are, respectively, equal

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Fig. 1

Figure 1: Fig. 1

to the concentration of free electrons and to the Debye length due to free and bound electrons $\left(N_1 = \frac{M_2}{b} n_1\right)$.

By means of the substitution $e^{-\psi/2} = y$, the problem is reduced to elliptic integrals, which makes it possible to represent the distribution of the potential in the photoelectret by means of the Weierstrass function $\wp(x/2)$ (5):

$$e^{-\psi/2} = 1 + \frac{\sqrt{f(1)}\,\wp'(x/2) + \frac{1}{2} f'(1)\{\wp(x/2) - \frac{1}{24} f''(1)\} + \frac{1}{24} f(1) f'''(1)}{2\{\wp(x/2) - \frac{1}{24} f''(1)\}^2 - \frac{1}{48} f(1) f^{\mathrm{IV}}(1)}, \quad (7)$$

where $f(y)/y^2$ is the expression under the radical in the integral after the substitution $e^{-\psi/2} = y$. The invariants of the function $\wp(x/2)$ are the expressions

$$g_2 = \frac{4}{L^4}\left\{1 + \frac{1}{3}\left[\frac{E_0^2 L^2}{4} - \operatorname{ch}\frac{V}{2}\right]^2\right\}; \quad (8)$$

$$g_3 = \frac{8}{3L^6}\left[\frac{E_0^2 L^2}{4} - \operatorname{ch}\frac{V}{2}\right]\left\{1 - \frac{1}{9}\left[\frac{E_0^2 L^2}{4} - \operatorname{ch}\frac{V}{2}\right]^2\right\}.$$

3. Making in the integrand of (5) the change of variables $x = \xi + l/2$; $\psi = -\varphi - V/2$ and noting that $\psi(l) = -V$, we arrive at the expression

$$\xi = \int_0^{\varphi} \left\{E_0^2 + \frac{4}{L^2}\left[\operatorname{ch}\varphi - \operatorname{ch}\frac{V}{2}\right]\right\}^{-1/2} d\varphi, \quad (9)$$

which shows that $\psi(l/2) = -V/2$ and that the distribution of the potential $\psi - \psi(l/2)$ is an odd function with respect to the middle of the electret, i.e.,

$$\psi(l - x) + \psi(x) = -V. \quad (10)$$

Therefore, E is an even function, and ρ is an odd function of $x - l/2$.

4. The difference between the field strength $E(x)$ in the crystal and the strength of the polarizing field E_1 is due to: a) dipole polarization and b) charge polarization: $E(x) = E_1/\chi + \widetilde{E}(x)$. Here $\widetilde{E}(x)$ is the field created by spatially separated charges that screen the external field E_1. E_1/χ follows the external field practically without inertia, whereas the relaxation time of the residual field $\widetilde{E}$ is of the order of $\chi/4\pi\sigma$ in the case of free charge carriers and can be very large ($\sim e^{\varepsilon/kT}$), if simultaneously

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with the removal of the polarizing field, illumination of the crystal ceases, and the displaced charges are localized at deep trapping levels ($\varepsilon \gg kT$). Let us show that, for strong charge polarization of the crystal, when outside the near-electrode regions $E \ll E_1/\varkappa$ and the residual potential difference on the short-circuited electret is close to the maximum possible ("strong photoelectrets"), the solution can be represented in elementary functions.

5. From $\psi(l/2) = -V/2$ and the condition of strong screening of the external field it follows that

$$E^2\left(\frac{l}{2}\right) = E_0^2 + \frac{4}{L^2}\left[1 - \operatorname{ch}\frac{V}{2}\right] \ll E_0^2,$$

i.e.

$$\operatorname{ch}\frac{V}{2} \simeq 1 + \frac{E_0^2 L^2}{4}.$$

Therefore,

$$V^* = \frac{2kT}{q}\ln\left\{1 + \frac{q^2 E_1^{*2} L^{*2}}{4k^2T^2\varkappa^2}\left[1 + \left(1 + \frac{8k^2T^2\varkappa^2}{q^2E_1^{*2}L^{*2}}\right)^{1/2}\right]\right\}, \tag{11}$$

where

$$L^* = \left[\varkappa kT/4\pi q^2\left(n_1^* + N_1^*\right)\right]^{1/2}. \tag{12}$$

6. For $\varkappa \to 0$, the function $x(\psi)$ is approximated by the integral

$$x = \int_\psi^0 \left[E_0^2 + \frac{2}{L^2}e^{V/2}(e^\psi - 1)\right]^{-1/2} d\psi, \tag{13}$$

i.e. the potential distribution in the pre-anode layer of the photoelectret has the form

$$\psi(x) = -2\ln\left\{\frac{1}{r}\cos[\arccos r - \gamma x]\right\}. \tag{14}$$

Correspondingly, in the near-cathode layer

$$\psi(x) = -V + 2\ln\left\{\frac{1}{r}\cos[\arccos r - \gamma(l - x)]\right\}. \tag{15}$$

Here

$$r = \left[\frac{(1 + 8/E_0^2L^2)^{1/2} + 4/E_0^2L^2 - 1}{(1 + 8/E_0^2L^2)^{1/2} + 4/E_0^2L^2 + 1}\right]^{1/2}; \tag{16}$$

$$\gamma = \frac{E_0}{2\sqrt{2}}\left[(1 + 8/E_0^2L^2)^{1/2} + 4/E_0^2L^2 - 1\right]^{1/2}.$$

Using formula (9), it can be shown that for $E_0L \gg 10$ the pre-anode region extends from $\psi = 0$ to $\psi = -V/2 + 1$, and the near-cathode region from $\psi = -V/2 - 1$ to $\psi = -V$. In this case the principal voltage drop occurs in the

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boundary regions, while in the remaining (quasineutral) volume of the crystal $\rho^* \simeq 0$ and $\Delta\psi^* \simeq 2kT/q$.

For $E_0L < 10$ the voltage drop in the boundary regions is small. It is comparable with the voltage drop in the bulk of the crystal, or even smaller than it. The condition $E_0L < 10$ is realized either when the polarizing field is small, or when the concentration of free electrons is so large that, in the thin boundary layers shielding the external field, the potential changes hardly at all (an almost metallic character of the charge polarization).

Real photoelectrets correspond to the case $E_0L \gg 10$. In this case

$$V = 4\ln\frac{E_0L}{\sqrt{2}}, \tag{17}$$

and the spatial distribution of the potential, field strength,

charge and concentrations in the near-anode region are described by the formulas

$$\psi = -2\ln\left\{\frac{E_0L}{2}\sin\left[\frac{2}{E_0L}+\frac{x}{L}\right]\right\}, \qquad E = \frac{2}{L}\operatorname{ctg}\left[\frac{2}{E_0L}+\frac{x}{L}\right];$$

$$\rho = -2n_1\left(1+\frac{M_2}{b}\right)\operatorname{cosec}^2\left[\frac{2}{E_0L}+\frac{x}{L}\right];$$

$$P = \frac{n_1}{2}\left(1+\frac{M_2}{b}\right)\sin^2\left[\frac{2}{E_0L}+\frac{x}{L}\right]; \tag{18}$$

$$n = 2n_1\operatorname{cosec}^2\left[\frac{2}{E_0L}+\frac{x}{L}\right]; \qquad N = 2\frac{M_2}{b}n_1\operatorname{cosec}^2\left[\frac{2}{E_0L}+\frac{x}{L}\right]$$

and in the near-cathode region by the formulas

$$\psi = -V + 2\ln\left\{\frac{E_0L}{2}\sin\left[\frac{2}{E_0L}+\frac{l-x}{L}\right]\right\}; \qquad E = \frac{2}{L}\operatorname{ctg}\left[\frac{2}{E_0L}+\frac{l-x}{L}\right];$$

$$\rho = 2n_1\left(1+\frac{M_2}{b}\right)\operatorname{cosec}^2\left[\frac{2}{E_0L}+\frac{l-x}{L}\right];$$

$$P = 2n_1\left(1+\frac{M_2}{b}\right)\operatorname{cosec}^2\left[\frac{2}{E_0L}+\frac{l-x}{L}\right]; \tag{19}$$

$$n = \frac{n_1}{2}\sin^2\left[\frac{2}{E_0L}+\frac{l-x}{L}\right]; \qquad N = \frac{M_2n_1}{2b}\sin^2\left[\frac{2}{E_0L}+\frac{l-x}{L}\right].$$

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From the conditions $\psi(x_1) = -V/2 + 1$ and $\psi(x_2) = -V/2 - 1$ we find that the boundaries of the near-anode and near-cathode regions are equal to

$$x_1^* = (l^* - x_2^*) = L^*(1.04 - 2kT/qE_0^* L^*) \simeq L^* \tag{20}$$

and are practically independent of the field strength.

The limits of applicability of the approximate solution obtained are determined by the conditions

$$\chi kT/\pi q^2 l^{*2} < n_1^* + N_1^* < E_1^{*2}/400\pi\chi kT. \tag{21}$$

7. The results obtained apply equally to hole photoelectrets, and also to thermoelectrets when, in the latter, the electret state is caused by the displacement of electrons or holes (but not ions) and is not complicated by the formation of charged layers at the boundaries of the microcrystallites ([6]).

In crystals with sufficiently deep donor and acceptor levels, the transition probabilities k_1^* and k_2^* under illumination are proportional to the light intensity I. From expressions (5) and (6) it follows that when $M_2/b \gg 1$ the parameter L, and consequently also $\psi(x)$, do not depend on the light intensity. But the inequality $M_2/b \gg 1$ is equivalent to the condition

$$n(x) \ll N(x) \qquad (0 \leq x \leq l). \tag{22}$$

Consequently, when inequality (22) is satisfied, the values $\rho(x)$, $E(x)$, $\psi(x)$, and V for $z \equiv It \to \infty$ do not depend on I, i.e., this inequality is the sole necessary and sufficient condition for the reciprocity law to hold at sufficiently large exposures.

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