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Abstract

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DIURNAL AND SEASONAL VARIATIONS OF THE FAR FIELD OF LONG RADIO WAVES

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To explain the diurnal variations of the far field, one must take into account the slow changes of the ionosphere along the radio-wave path, i.e., assume that $\|\varepsilon\| = f(r, \mu\theta, \mu\varphi, \mu t)$, where μ is a small parameter. Expanding the solution of the problem in powers of μ, we obtain for the field E_r at a point P on the Earth ($r = a$), produced by a Hertz dipole at a point O on the Earth, an expression in the form of a spectrum of modulated normal waves with wave numbers calculated by formula (10) from (^{2}) for $\|\varepsilon\|$, "frozen" with respect to the coordinates θ, φ, t at the given point on the path:

$$E_r \cong \sqrt{\frac{W}{\sin\theta}} e^{i\pi/4} \sum_l n_l(O) n_l(P) \exp\left[i \int_0^P \nu_l ds\right] \frac{\mu\text{V}}{\text{m}}, \qquad (1)$$

where W is the power in kilowatts, $l = j = 0, 1, 2, \dots, n$ and $l = k = 1, 2, \dots, m$.

Thus, $\chi_{j;k} = \chi_{j;k}(\mu\theta, \mu\varphi, \mu t)$ vary slowly along the path, and the integrals in (1) are taken along ray lines connecting the points O and P in a two-dimensional medium with refractive index $\nu = \sqrt{\chi}$. The coefficients $n(O)$ and $n(P)$ are computed from the eigenfunctions

$$\left| \begin{matrix} Y_j(r, \mu\theta, \mu\varphi, \mu t) \\ Z_j(r, \mu\theta, \mu\varphi, \mu t) \end{matrix} \right|$$

at the points O and P.

Our task is to find such functions $N_e(r; \mu\theta, \mu\varphi, \mu t)$ and $\nu_{\text{eff}}(r, \mu, \theta, \mu\varphi, \mu t)$ as provide the best agreement of all known experimental data on the fields $E(a, \theta, \varphi t)$ and $H(a, \theta, \varphi, t)$. Restricting ourselves to field variations and to middle latitudes, we shall assume that: 1) the rays are only slightly curved, and they may be replaced by geodesic lines $\widehat{OP}$, putting $ds = d\theta$; 2) ν_{eff} does not depend on θ, φ and t and is defined according to Nicolet (^{3}); 3) in the daytime N_e depends only on the solar elevation angle χ, and at night on the time τ elapsed since sunset in the lower ionosphere; 4) in accordance with the conclusions of work (^{3}), we replace $N_e(r)$ by a single-step curve and, taking into account that $\omega_H^2 \ll \nu_{\text{eff}}^2$, describe the daytime layer by the parameters $\sigma_0(\mu\mho/\text{m}) = 2.8 \cdot 10^4 N_e^0/\nu_{\text{eff}}^2$ and h_0, and at night by N_e^0, ν_{eff}^0 and h_0.

On the basis of these simplifications, world maps of the lower ionosphere were constructed for different seasons, with zones of equal χ and τ indicated (Fig. 1). The ionospheric parameters plotted on them were found by the method of mixed initial data (m.i.d.) (3) from the diurnal variations $|E_r(t)|$ of the English stations GBR (Rugby, 16 kc) and GBZ (Criggion, 19.4 kc), which were compared by us in Moscow ($D \cong 2500$ km) during 1950-1958 and then averaged with a 2-week interval. One of 24 transparent ionization maps is superimposed on the map of paths with the $\widehat{OP}$’ s plotted on it. The ionization distribution on the path at any moment of the day is found by rotating the upper map relative to the lower one. From these data one finds the parameters of the normal waves $n(O)$, $n(P)$ and ν entering into (1), and computes the field E_r; the nighttime values of n and ν are determined by linear interpolation in h_0 between $h_{0,\max} = 88.5$ km (see Fig. 3 (3)) and h_0 at $\tau = 0$. From Fig. 1 it is seen that $h_0(\tau)$ depends strongly on the season. The rise of the lower layer after sunset occurs most rapidly in summer and most slowly in winter, but, ne-

depending on τ, it ends at heights $h_0 \simeq 89$—91 km with the distribution N_e^{n}, shown in Fig. 1 (3). The times of sunrise and sunset fluctuate by 15-20 min, depending on the ozone content at heights of 40-60 km, which delays the active solar radiation during these periods. The emergence of the lower layers occurs in two stages: a rapid descent (20-30 min) from $h_{0,\max}$ to $h_0 = 65$—70 km (at the speed with which the sun’ s ray advances into the shadow) in the form of a wedge-shaped profile of layer C (Fig. 1 (3)), and then a slow transition to the normal profile N_e^{d} over several hours due to the formation of layer D. This mechanism explains the different behavior of the phase of the far and near fields (4,5), but the maps in Fig. 1 are suitable only for the far field, since they take into account the total effect of layers C and D. Naturally, $|E_r(t)|$ in Fig. 2 coincides with the experimental curve used in the method of the s. i. d. The phase of GBR was not measured, but we are certain that the calculated curve $\arg E_r(t)$ predicts the phase behavior with an accuracy of 10-20%. The field dips A and B in Fig. 2 are caused by the fall of the $|n|$ wave TH_1 as h_0 increases. The nighttime dip of $|E_r(t)|$ is caused by interference cancellation of the waves TH_1 and TH_2, as is evident from the vector diagrams in Fig. 2. It is observed only for $\theta \simeq 0.4$ rad.

Figure 4 gives the classical picture of the mysterious di—

Fig. 1. Ionization maps of the Northern Hemisphere. a—equinox (spring), b —summer solstice, and v—winter solstice. (For the autumn equinox, the spring map is taken with summer values of σ.) On map a the GBR—Moscow path is plotted (h in kilometers, σ in micromhos per meter, time in hours).

Fig. 2

Figure 1: Fig. 2

Fig. 3

Figure 2: Fig. 3

nocturnal oscillations $|E_r(t)|$ of stations WQL (USA, Long Island, 17.13 kHz, $\theta = 0.86$, $W = 12$ kW), 2XS (same place, 57 kHz, $W = 21$ kW), received in September 1923 in New Southgate (England) by Espenschied, Anderson, and Bailey (^{7}). The drop in the 2XS field in the evening is caused by a change of the leading waves during the rise of layer D: TH_2—TH_3 in the daytime and TH_4—TH_5 at night. Judging from (^{8}), the morning drop of 2XS also exists.

Fig. 2. Diurnal variation of the amplitude and phase of GBR in Moscow, calculated with the aid of the map in Fig. 1a, and the experimental curve $|E_r|$, recorded on 19 III 1955.

Long-term measurements of $|E_r|$ by Austin at the U.S. Bureau of Standards are systematically understated by almost a factor of two, which also affected the Austin-Cohen phenomenological formula (^{9}). According to the theory (^{14}), the attenuation coefficients of the leading waves β_j over land should be increased by $\Delta\beta_j = 0.3$—0.5, which explains the shadow effects of continents observed in the reception of long waves on ships (^{10}, ^{11}). We have carried out calculations of E_r for the U.S. stations NSS (15.1 kHz), NPM (16.4 kHz), and NPG (18.3 kHz), received in Moscow, and refined the ionization maps of the polar region. It turned out that in winter, despite the absence of solar radiation, there exists there a lower ionospheric layer of sufficient intensity. Apparently, it is produced by soft X-radiation arising during the braking of electrons of the radiation belt. This phenomenon (^{12}) explains nocturnal disturbances of the lower ionosphere not only in the polar region but also in middle latitudes, lasting from several days to 2-3 weeks,

Fig. 3. Diurnal variation of the phase of GBR in Cambridge (USA), calculated from the ionization map for 15 VII, and experimental curves obtained by Pierce in 1954 from 16 to 18 VII.

when h_0 falls to 70-80 km. The daytime layer undergoes perturbations mainly because of hard X-ray radiation ($\lambda < 10$ Å) arising in active regions of the solar corona $^{(13)}$. As processing of the daytime $|E_r|$ of old radio stations has shown $^{(11,7,8)}$, the mean values of h_0 and σ_0 have an 11-year period of oscillation with ranges of 2-4 km and 0.2-0.5 $\mu\mho$/m, respectively.

Fig. 4. Diurnal variation of the effective field magnitude $|\overline{E}_r|$ for WQL and 2XS, calculated (dashed line) from the autumnal-equinox map, and experimental effective values obtained at New Southgate (England) on 23 IX 1923. $^{(7)}$

Layer C and the lower part of layer D play an essential role in the absorption of

Fig. 4. Diurnal variation of the effective field magnitude $|\overline{E}_r|$ for WQL and 2XS, calculated (dashed line) from the autumnal-equinox map, and experimental effective values obtained at New-Southgate (England) on 23 IX 1923. (7)

Figure 3: Fig. 4. Diurnal variation of the effective field magnitude $|\overline{E}_r|$ for WQL and 2XS, calculated (dashed line) from the autumnal-equinox map, and experimental effective values obtained at New-Southgate (England) on 23 IX 1923. (7)

short radio waves in the daytime. Without taking sphericity into account, the discrepancy between theory and experiment for $|E_r|$ reaches several hundred percent at $D > 2000$ km, while the mean calculated phase velocity v_{av} is always greater than $c = 3 \cdot 10^5$ km/sec, although from Fig. 3 (3) it follows that, with sphericity taken into account, $v_{\text{av}} > c$ for $f > 15$–20 kc in the daytime and for $f > 10$ kc at night. If desired (15–17), these discrepancies can be reduced by varying the ionospheric parameters h_0, σ_0 as functions of D and f, but the resulting set of values $h_0(\theta, f)$ and $\sigma_0(\theta, f)$ can be regarded only as a table which, in encoded form, contains information already known about the radio-wave field used in calculating h_0 and σ_0, while the theory turns into a rule for decoding this table.

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Note: Figure translations are in progress. See original paper for figures.