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Abstract

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ON THE β-DECAY OF STRONGLY DEFORMED NUCLEI

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On the basis of the mathematical methods developed by N. N. Bogolyubov, in ([1]) the superconducting model of the nucleus was formulated, and in ([2,3]) studies were carried out of a number of properties of strongly deformed nuclei. The most important distinction of the superconducting model of the nucleus from the pair correlations considered earlier ([4,5]) is the allowance for changes in the superconducting properties of the nucleus in the transition from ground to excited states. In the superconducting model of the nucleus, all excited states of a system consisting of N particles are assigned to the given system of N particles, i.e., the number of particles is conserved on the average, which is connected with the approximate method of studying the many-body problem; moreover, as shown in ([2,3]), the error does not exceed 6%. Let us note that in the original formulation of superconducting properties ([4,5]), the number of particles was not conserved even on the average. Thus, among the excited states of a system whose ground state consisted of an even number N of particles, there were levels corresponding to $N-2$, N, $N+2$ particles, while in the excitation spectrum of a system consisting of an odd number N' of particles there were mixed states corresponding to $N'-2$, N' or N', $N'+2$ particles. In this connection, it was possible to make only very rough estimates of the influence of pair correlations on β-decay ([6]).

On the basis of the superconducting model, it is possible to take into account changes in the structure of the nucleus as a many-body system in β- and γ-transitions of quite definite nuclei. In ([1,2]) it is shown that the role of superconducting corrections to the probabilities of β- and γ-transitions in strongly deformed nuclei, taking into account the rearrangement of the nucleus, is in many cases very important.

In the present work we formulate general rules for constructing corrections to β-decay associated with the superconductivity of the ground and excited states; we carry out, in addition to Alaga' s selection rules, a classification of the probabilities of β-decay of strongly deformed nuclei; and we investigate the role of superconducting corrections by analyzing the values of $\log ft$ for β-transitions between identical pairs of single-particle states in different nuclei.

The matrix element describing the β-decay of a complex nucleus will be written symbolically as

$$M \sim \Psi^*_{2n_N} \Psi^*_{2n_Z+1}(s_2) \sum_{\nu,\nu'} \langle \nu \mid \Gamma \mid \nu' \rangle a^+_\nu b_{\nu'} \Psi'_{2n_Z} \Psi'_{2n_N+1}(s_1) = \langle s_2 \mid \Gamma \mid s_1 \rangle L; \tag{1}$$

here $\langle s_2 \mid \Gamma \mid s_1 \rangle$ is the single-particle transition matrix element, and

$$L = (\Psi^*_{2n_N} \Psi'_{2n_N})(\Psi^*_{2n_Z} \Psi'_{2n_Z}),$$

where Ψ_N is the wave function of the system of N particles. The values of ft characterizing β-decay will then be obtained in the form

$$ft = \frac{\text{const}}{|\langle s_2 \mid \Gamma \mid s_1 \rangle|^2} L^{-2}, \tag{2}$$

where L^2 is represented in the form $L^2 = R_Z R_N$. The quantities R_Z and R_N describe the rearrangement of the nucleus in a β-transition, the first referring to the rearrangement of the proton system and the second to that of the neutron system.

Let us give general rules for constructing the corrections R (i.e., R_Z or R_N) on the basis of the superconducting model of the nucleus, using the wave functions, equations, and notation given in ([2]). We shall consider the proton and neutron systems independently. We shall find R for β-decays with any number of quasiparticles participating in the initial and final states, except for those cases in which there are two quasiparticles on one and the same level. We write R in the form

$$R = \gamma \prod_{s \neq f_1 \cdots f_k} (u_s u'_s + v_s v'_s)^2; \tag{3}$$

the functions u_s, v_s refer to the initial state, and u'_s, v'_s to the final state.

In the product $\prod_{s \neq f_1 \cdots f_k} (u_s u'_s + v_s v'_s)^2$ there are no factors corresponding to the levels on which there are quasiparticles, and the product itself is the closer to unity the more similar the superconducting properties of the initial and final states. In the formulation of ([4,5]) of pair correlations this product is equal to unity. Further, if the number of paired particles in the initial and final states is the same, then $\gamma = u_f^2$, while if the number of paired nucleons changes in the decay process, then $\gamma = v_f^2$, where f refers to the level on which a quasiparticle has disappeared or appeared. The functions u_f^2 and v_f^2 in (3) characterize the superconducting properties of the system with the smaller number of quasiparticles.

Consider the case in which the constant of the pair interaction G tends to zero, i.e., when the superconducting model goes over into the independent-particle model. Then the correction R takes one of two values: $R = 1$ or $R = 0$, with $R = 1$ corresponding to the case in which β-decay proceeds without changing the position of all nucleons except one, while in the case $R = 0$ the β-decay proceeds with a change in the position of more than one nucleon in the independent-particle model. For a β-decay in which the number of pairs remains unchanged, $R = 1$ for particle transitions and $R = 0$ for hole transitions; and for a β-decay in which the number of pairs changes by one, $R = 1$ for hole transitions and $R = 0$ for particle transitions. We shall call particle transitions those in which a quasiparticle disappears or appears on one-particle levels f whose energy is greater than the value λ (which plays the role of the chemical potential) belonging to the system with the smaller number of quasiparticles. For hole transitions the energies of the one-particle levels f are less than the value λ.

Let us carry out an additional classification of β-decays of strongly deformed complex nuclei, in comparison with the selection rules of Alaga formulated in ([7]); namely, let us divide all β-transitions into three groups:

I group $\quad R(G = 0) = 1, \quad 0 < R(G \neq 0) < 1.$

II group $\quad R(G = 0) = 0, \quad 0 < R(G \neq 0) < 1.$

III group $\quad R(G = 0) = 0, \quad R(G \neq 0) = 0.$

To group I we assign: a) those β-decays for which the initial and final states are the ground states of the system; b) particle transitions when the number of pairs is unchanged; c) hole transitions when the number of pairs changes by one.

To group II we assign: a) hole transitions when the number of pairs of particles is unchanged; b) particle transitions in the case in which the number of pairs of particles changes by one. For β-decays assigned to group II, the superconducting model gives transition probabilities different from zero, whereas these transitions are strictly forbidden in the independent-particle model. We note that the corrections R calculated on the basis of the superconducting model of the nucleus and assigned

to groups I and II, which are associated with β-transitions to low-lying excited states of nuclei (~0.2 MeV), are equal to one another in order of magnitude; in the case of transitions to highly excited states (~1 MeV and above) a significant difference appears between them.

The analysis of the experimental data shows that there are more than two dozen firmly established β-transitions assigned to group II. The discovery of β-transitions assigned to group II testifies to the advantage of the superfluid nuclear model over any model of independent particles and is yet another confirmation of the existence of an important short-range pairing interaction.

If groups I and II include those β-decays in which only one quasiparticle disappears or appears in the proton (neutron) systems, while the positions of the remaining quasiparticles remain unchanged, then to group III we shall assign: a) transitions with a change in the number of quasiparticles of the proton (neutron) system by more than one; b) transitions in which, along with a change in the number of quasiparticles by one, the position of other quasiparticles changes.

The superfluid nuclear model is a model of independent quasiparticles; therefore transitions connected with a rearrangement of quasiparticles are strictly forbidden in it. It is of interest to investigate the degree of forbiddenness of transitions assigned to group III. It is very likely that they are strongly retarded in comparison with transitions assigned to groups I and II. To determine the degree of forbiddenness of β-transitions assigned to group III, one should experimentally investigate the possibility of the appearance of β-decays of one-quasiparticle states of an odd system into such two-quasiparticle excited states of an even system that all three quasiparticles are on different single-particle levels.

To reveal the role of superfluid corrections, let us analyze the values of $\log ft$ for β-transitions between pairs of identical single-particle states in different nuclei. We note that with such an approach the influence of the single-particle matrix element $\langle s_1|\Gamma|s_2\rangle$ on the relative values of $\log ft$ is not completely excluded, since in passing from one nucleus to another the mean field changes somewhat. Table 1 gives three series of β-decays of this type.

Table 1

Initial and final states	β-transition	Classification	$(\log ft)_{\text{exp}}$	$R_Z R_N$	$(\log ft)_{\text{rel}}$
$s_Z = \{5/2-[532]\} s_N = \{3/2-[521]\}$	$Tb^{159} \leftarrow Gd^{159}$	*ah* II I	6.7 (7)	0.07	6.7
$s_Z = \{5/2-[532]\} s_N = \{3/2-[521]\}$	$Ho^{161} \leftarrow Er^{161}$	*ah* I I	5.6 (9)	0.35	6.0
$s_Z = \{3/2-[521]\} s_N = \{1/2-[631]\}$	$Np^{237} \leftarrow U^{237}$	1*u* I I	6.0 (7)	0.5	6.0

Initial and final states	β-transition	Classification	$(\log ft)_{\mathrm{exp}}$	$R_Z R_N$	$(\log ft)_{\mathrm{rel}}$
$s_Z = \{3/2 - [521]\} s_N = \{1/2 - [631]\}$	$Am^{241} \leftarrow Cm^{241}$	$1u$ II I	7.4 ([7])	0.1	6.7
$s_Z = \{3/2 - [521]\} s_N = \{1/2 - [631]\}$	$Bk^{245} \rightarrow Cm^{245}$	$1u$ I II	~7.4 ([7])	0.05	7.0
$s_Z = \{5/2 - [642]\} s_N = \{7/2 - [743]\}$	$Np^{239} \rightarrow Pu^{239}$	$1u$ I I	6.5 ([7])	0.82	6.5
$s_Z = \{5/2 - [642]\} s_N = \{7/2 - [743]\}$	$Np^{237} \leftarrow Pu^{237}$	$1u$ I I	6.8 ([7])	0.36	6.9
$s_Z = \{5/2 - [642]\} s_N = \{7/2 - [743]\}$	$Np \rightarrow U^{235}$	$1u$ I I	~7.5 ([7])	0.24	7.1

β-decays. In the column giving the classification of β-decay, first the group of the proton transition is indicated, and then that of the neutron transition. The table also gives the values $(\log ft)_{\mathrm{rel}}$, calculated relative to the first transition in the given series of transitions. It follows from Table 1 that allowance for pairing correlations largely explains the retardation of the process $Gd^{159} \rightarrow Tb^{159}$, where the proton transition belongs to group II and the neutron transition to group I, in comparison with the process $Er^{161} \rightarrow Ho^{161}$, where both transitions belong to group I. The influence of superfluidity on β-decay is more fully illustrated in transitions between the states $3/2 - [521]$ and $1/2 + [631]$, where there are both proton and neut-

decays assigned to group II. All three transitions between the states ${}^{5}/{}_{2} + [642]$ and ${}^{7}/{}_{2} - [743]$ belong to group I; however, taking pairing correlations into account leads to a change in $\log ft$, which improves agreement with the experimental data. Let us note that a certain discrepancy in the case of $Np^{235} \rightarrow U^{235}$ is connected with a change in the average field, as is seen from the spectra of single-particle levels of odd nuclei.

The classification of β-transitions presented here and the inclusion of the calculated superfluid corrections prove useful in analyzing the β-decays of even nuclei. In ([10]) it is shown that single-particle transitions proceed at approximately the same rates in the β-decays of odd and even nuclei. Let us determine the single-particle matrix element from the β-decay of an odd nucleus and use it to calculate $\log ft$ on the basis of the superfluid model for the corresponding β-transition in an even nucleus. Indeed, determining the single-particle matrix element of the β-decay of W^{187} from the state ${}^3/_2-[512]$ to the state ${}^5/_2+[402]$ of Re^{187}, knowing that $(\log ft)_{\text{exp}} = 7.9$ ([11]), $R = 0.33$, and using it, we obtain $(\log ft)_{\text{theor}} = 7.8$ for the corresponding transition $Re^{186} \to Os^{186}$ and $(\log ft)_{\text{theor}} = 7.94$ for the transition $Re^{188} \to Os^{188}$ between ground states, which is in good agreement with the experimental data ([10]), $(\log ft)_{\text{exp}} = 7.7$ and $(\log ft)_{\text{exp}} = 8.0$, respectively.

Similar rules have been found for constructing superfluid corrections to electromagnetic transitions, for which an additional classification has also been introduced.

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Note: Figure translations are in progress. See original paper for figures.