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**Abstract**

**Full Text**

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## On indexials of finite groups

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§ 1. In papers ([1−4]) we proposed and developed the method of "indexials" for finding subgroups in finite groups, and with its aid a general theorem on the existence of subgroups was obtained, embracing as special cases many principal earlier results in this area.

In the present paper, in order to strengthen this theorem, the notions of a regular indexial, an extension of an indexial, and a measure of solvability of a finite group are introduced; with their aid, assertions 5) and 6) of the theorem of the article ([4]) are refined, and Theorem 2 of ([1,2]) is generalized.

The main result of the present paper is Theorem 5.

§ 2. We shall use the definitions and notation introduced in ([3]), as well as the following:

1) The sequence $\omega$ and the factor groups $\mathfrak{F}_i/\mathfrak{G}_i,\ i\in\omega$, from article ([3]) will be called respectively the **basis** and the **factors** of the indexial $(h)_{R,f}$.
2) Let $I$ be the sequence of indices of some composition (chief) series of a finite group $\mathfrak{G}$, and let $I_{\text{pr}}$ be the subsequence of all those indices from $I$ which are prime numbers; then the number $\bar{I}_{\text{pr}}$ (for the meaning of the symbol $\bar{I}_{\text{pr}}$, see ([3])) will be called the **measure of solvability** (**strong solvability**) of the group $\mathfrak{G}$.
3) An indexial $(h)_{R,f}$ will be called **regular** if it satisfies the following two requirements: a) the group $\mathfrak{G}$ has at least one subgroup $\mathfrak{H}$ of order $h$, contained in $\mathfrak{G}_{\beta-1}$; b) for each $i\in\omega$ one has $\mathfrak{F}_i=[\mathfrak{H}\cap\mathfrak{G}_{i-1}]\mathfrak{G}_i$. In this case the subgroup $\mathfrak{H}$ will be called a **suitable subgroup of the indexial** (for the indexial) $(h)$.
4) Let $\omega$ be a common basis; $f_\beta,\ f_{\beta+1},\dots,f_\omega$ and $\varphi_\beta=c_\beta f_\beta,\ \varphi_{\beta+1}=c_{\beta+1}f_{\beta+1},\dots,\ \varphi_\omega=c_\omega f_\omega$ ($c_\beta,c_{\beta+1},\dots,c_\omega$ are natural numbers, of which $c_\beta=1$) are the components respectively of the indexials $(h)_{R,f}$ and $(ch)_{R,\varphi}$, $c=c_\beta c_{\beta+1}\cdots c_\omega$, of the group $\mathfrak{G}$. If for each $i\in\omega$ the factor $\mathfrak{C}_i/\mathfrak{G}_i$ of order $c_i f_i$ of the indexial $(ch)_{R,\varphi}$ is an extension in $\mathfrak{G}_{i-1}/\mathfrak{G}_i$ of the corresponding factor $\mathfrak{F}_i/\mathfrak{G}_i$ of the indexial $(h)_{R,f}$ by means of a group $\mathfrak{C}_i^*$ of order $c_i$, then the indexial $(ch)_{R,\varphi}$ will be called an **extension of the indexial** $(h)_{R,f}$. The number $c$ will be called the **measure of extension** of the indexial $(h)$.

5) An extension $(ch)_{R,\varphi}$ of the indexial $(h)_{R,f}$ will be called: a) **solvable** (**special**) if each group $\mathfrak{C}_i^*$, $i \in \omega$, is solvable (special); b) **arithmetically closed** if $\Pi(c_i) \subseteq \Pi(f_\beta f_{\beta+1} \dots f_{i-1})$ for all $i = \beta+1,\ \beta+2, \dots, \omega$; c) **specially arithmetically closed** (s.a.c.) if $(ch)_{R,\varphi}$ is a special and arithmetically closed extension of the indexial $(h)_{R,f}$; d) **regular** if $(ch)_{R,\varphi}$ is a regular indexial; e) **regular s.a.c.** if $(ch)_{R,\varphi}$ is a regular and s.a.c. extension of the indexial $(h)_{R,f}$; f) **trivially s.a.c.** if $c_i = 1$ for each $i \in \omega$.

6) An indexial admitting no s.a.z. extensions other than the trivial one shall be called s.a.z. **maximal**.

7) If $f_i^*$, $i \in \omega$, is the solubility measure of the factor $\mathfrak{F}_i/\mathfrak{G}_i$ of the indexial $(h)_{R,f}$, then

$$h^* = f_\beta^* f_{\beta+1}^* \cdots f_\omega^*$$

shall be called the **solubility measure** of $(h)_{R,f}$.

3. We present the results obtained by us.

**Theorem 1.** If $(h)_{R,f}$ is a proper indexial and $\mathfrak{H}$ is its corresponding subgroup, then the factors of the series

$$\mathfrak{H} = \mathfrak{H} \cap \mathfrak{G}_{\beta-1} \supseteq \mathfrak{H} \cap \mathfrak{G}_\beta \supseteq \cdots \supseteq \mathfrak{H} \cap \mathfrak{G}_\omega = \mathfrak{E}$$

of normal divisors of $\mathfrak{H}$ are respectively isomorphic to the factors of the indexial $(h)_{R,f}$.

**Theorem 2.** If $(h)_{R,f}$ is a proper indexial and $\mathfrak{H}$ is its corresponding subgroup, then the solubility measure of $\mathfrak{H}$ is equal to the solubility measure of $(h)_{R,f}$.

**Theorem 3.** If $(ch)_{R,\varphi}$ is a soluble extension of the indexial $(h)_{R,f}$, then the solubility measure of $(ch)_{R,\varphi}$ is equal to $ch^*$, where $c$ is the measure of the extension and $h^*$ is the solubility measure of $(h)_{R,f}$.

**Theorem 4.** If $(ch)_{R,\varphi}$ is a proper and soluble extension of the indexial $(h)_{R,f}$, then the solubility measure of the corresponding subgroup $\mathfrak{H}$ of the indexial $(ch)_{R,\varphi}$ is equal to $ch^*$, where $c$ is the measure of the extension and $h^*$ is the solubility measure of $(h)_{R,f}$.

**Theorem 5.** Every indexial has at least one proper s.a.z. extension.

**Theorem 6.** Every s.a.z. maximal indexial is proper.

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*Note: Figure translations are in progress. See original paper for figures.*