PENETRATION OF AN ARBITRARY PRESSURE INTO A COMPRESSIBLE LIQUID IN THE ISENTROPIC CASE

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Abstract

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PENETRATION OF AN ARBITRARY PRESSURE INTO A COMPRESSIBLE LIQUID IN THE ISENTROPIC CASE

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Let, at some point O of a compressible liquid, a certain pressure arise, which propagates according to an arbitrary law symmetric with respect to the point O. The equation of state of the liquid is a polytrope

$$P = B(S)\left[\left(\frac{\rho}{\rho_0}\right)^n - 1\right], \tag{1}$$

where P is the pressure; $B(S)$ is a slowly varying function; ρ is the density; S is the entropy. For pressures of the order of 1000 kg/cm^2, the motion of the liquid may be assumed isentropic. The motion of the liquid has axial symmetry.

Choose the axis Ox along the surface of the liquid, and direct the axis Oy into the depth of the liquid. On the surface $y = 0$ we have the boundary condition:

$$P(x,0,t) = \begin{cases} P_1(x,t), & x < R(t), \\ 0, & x > R(t), \end{cases} \tag{2}$$

where t is the time from the beginning of the motion; $x = R(t)$ is the radius of the pressure front on the surface.

As shown in ([1]), for P of the order of 1000 kg/cm^2 the quantity P/Bn will be small, and the elementary waves arising at the points $x = x'$ of the surface at the instant $t = t'$ may be regarded as Riemann waves

$$(x - x' - u)^2 + (y - v)^2 = a_1^2(x', t')(t - t')^2, \tag{3}$$

where u, v are the components of the velocity of the particles on the surface along the axes Ox and Oy; $a_1(x', t')$ is the speed of sound at the point $x = x'$ at the instant $t = t'$, and, according to (1),

$$a_1(x', t') = a_0\left[1 + \frac{n-1}{Bn} P_1(x', t')\right]; \tag{4}$$

a_0 is the speed of sound in the liquid at rest.

In expression (3) we neglect u, v in order to simplify the exposition, and write the equations of the surfaces of constant pressures and speeds of sound (level surfaces). The equations of the level surfaces are found as the envelopes of (3) for $a_1(x', t') = \text{const}$. Finally we have:

$$(x - x')^2 + y^2 = a_1^2(x', t')(t - t')^2, \tag{5}$$

$$(x - x')\frac{\partial x'}{\partial t'} = a_1^2(x', t')(t - t'),$$

where $\partial x/\partial t'$ for constant $a_1(x', t')$ has the form

$$\frac{\partial x'}{\partial t'} = -\frac{\partial a_1(x', t')/\partial t'}{\partial a_1(x', t')/\partial x'}.$$

Equations (5) serve to determine x' and t' as functions of x, y, t and to determine the pressure $P(x, y, t) = P_1(x', t')$ at points behind the shock wave, which in our approximation does not affect the flow behind it. To determine the shock wave we use the approximate formula for the shock-wave velocity (Pfrim' s formula)

$$D = \frac{(n-3)a_0 + (n+1)a_1(x', t')}{2(n-1)} = \frac{\partial y/\partial t}{\sqrt{1 + (\partial y/\partial x)^2}}.$$

The last equation, after substituting into it x' and t' as functions of x, y, t from (5), with the boundary condition

$$y\big|_{x=R(t)} = 0$$

serves to determine $y = f(x, t)$, the equation of the shock wave. After determining successive values of y for given t and x, one can determine x' and t' from (5), and then, from $P = P_1(x', t')$, determine the pressure at points of the shock wave.

However, it is simpler to substitute y through t' from (5) into the equation for the shock-wave velocity and obtain a differential equation for $t'(x, t)$. For a number of values of the time t and the coordinates x for the wave, computations were carried out; their results are given in Table 1.

Table 1

$t = 0.0155$ sec., $R'(t) = 3217.7$ m/sec

x	91.72	76.98	65.21	54.50
y	0	6.27	9.35	13.53
P_1, kg/cm^2	106.71	168.91	199.70	228.56

For water we have: $n = 7$, $B = 3045$ kg/cm^2, $a_0 = 1540$ m/sec. For the boundary pressure we take a certain approximation to the true pressures in an explosion in the atmosphere (a sharply decreasing boundary pressure)

$$P_1(x', t') = 1.2048 \left[\left(\frac{R'(t')}{340} \right)^2 - 1 \right] f \left[\frac{x'}{R(t')} \right],$$

$$f \left[\frac{x'}{R(t')} \right] = 8.729 - 7.481 \frac{x'}{R(t')} - 7.284 \sqrt{\left[1.153 - \frac{x'}{R(t')} \right]^2 - 0.022241}\,.$$

For the given parameter values, the shock wave will be shallow, and in the formula for the shock-wave velocity one may neglect $(\partial y/\partial x)^2$ in comparison with unity.

In the case of the self-similar boundary condition (2)

$$P_1(x', t') = P_a f \left(\frac{x'}{t'} \right), \qquad R'(t) = V,$$

where P_a and V are constants.

The level surfaces (5) are written in the form

$$\eta = \frac{\xi' - \xi}{\sqrt{\xi'^2 / a_1^2(\xi') - 1}}, \tag{6}$$

where

$$\xi = x/t, \qquad \eta = y/t, \qquad \xi' = x'/t'.$$

As is seen, the level surfaces (6) coincide with the level surfaces for a simple wave. To determine the shock wave, one obtains an ordinary differential equation. In particular, for the linear boundary condition $a_1(\xi') = A\xi'$, we obtain on the shock wave

$$\frac{a(\xi, \eta)}{A} = V \left(\frac{\xi}{V} \right)^m - \frac{1}{2Am} \frac{n-3}{n-1} a_0 \sin\varphi \left[\left(\frac{\xi}{V} \right)^m - 1 \right], \tag{7}$$

where

$$m = -\frac{n+1}{n-1}\frac{\sin\varphi}{2} + 1.$$

For the value $a_0/V = 1/6$ and $A = 1/3$, calculations were made by formula (7), and the results are given in Table 2.

Table 2

ξ/V	0.95	0.9	0.8	0.7	0.4
P, kg/cm^2	6863	6610	6083	5511	3400

For subsonic propagation, at large times one may determine the interaction of the shock wave with rarefaction waves from the surface by means of the theory of reflection of a rectilinear shock wave from a surface (1).

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Note: Figure translations are in progress. See original paper for figures.