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Abstract

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ON THE PECULIARITIES OF LIGHT ABSORPTION IN AN INHOMOGENEOUS PLASMA

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I. A number of studies in recent years confirm the possibility of a quite correct description of the complex set of phenomena occurring in an inhomogeneous high-temperature plasma (arc, pulsed discharge, hollow cathode, wire explosion, etc.) on the basis of the models of axially symmetric radiators of Bartels ([1]) and Cowan–Dieke ([2]). The degree of generality and the interrelation of these models were considered in ([3]).

It is natural to attempt to generalize, to the case of an inhomogeneous absorbing layer, the practically important dependences of the equivalent line width on optical density, whose graphs are called "curves of growth." The calculation of the indicated dependences for a homogeneous layer, denoted below by $y_1(x)$, was the subject of ([4–6]); here the index 1 corresponds to the equality $n = q = 1$, where n and q are the inhomogeneity parameters of Cowan–Dieke and Bartels ([3]).

It is convenient to write the general solution of the problem in the form

$$y_q(x) = y_1(x)\,[1 - 0.01F(x; q, a)], \tag{1}$$

where $F(x; q, a)$ is a deviation function having asymptotes at $x \to \infty$ and at values of the Voigt parameter a different from zero ([7]). Bearing in mind the relation established by us earlier between $F(x; q, a)$ and the reabsorption function $S(x; q, a)$ ([7]), and taking into account the assumptions underlying the Bartels model, we write:

$$S(x; q, a) = \int_{-\infty}^{+\infty} \mu e^{-\mu} \left\{ 1 + \sum_{k=1}^{\infty} \frac{\mu^{2k} f(k, q)}{(2k+1)!} \right\} d\omega; \tag{2}$$

$$\mu = \frac{0.0472 x e^{-a^2} a}{[1 - \operatorname{erf}(a)]\pi} \int_{-\infty}^{+\infty} \frac{e^{-y^2}\, dy}{a^2 + (\omega - y)^2}, \qquad 1 \le f(k = i, q) \le i. \tag{2a}$$

To obtain a family of "curves of growth" corresponding to some value of q, it is necessary to fix $f(k,q)$; x will then be equal to the quantity whose logarithm is plotted along the abscissa axis of the usual Held—Penner graph. Setting, in particular,

$$f(k,q) = \frac{1}{\lg q} \sum_{j=2}^{2k+1} \lg \frac{j}{n+j-1}, \qquad q = \frac{6}{(n+2)(n+1)}, \tag{3}$$

we arrive at the expression $S_c(x;n,a)$, corresponding to the Cowan—Dieke model,

$$S_c(x;n,a) = \int_{-\infty}^{+\infty} \mu e^{-\mu} \sum_{k=0}^{\infty} \frac{\mu^{2k}\Gamma(n+1)}{\Gamma(n+2k+1)}\, d\omega. \tag{4}$$

For the calculation, both the direct method of graphical integration using tables of the Voigt function ([8,9]), which enters the expression for μ, and the method of term-by-term integration of (2) or (4) were used. The latter is carried out by first expanding the exponential factor in a series and writing the Voigt function with the aid of the Stokes transformation ([6]):

$$\frac{a}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-y^2}\,dy}{a^2+(\omega-y)^2} = (\cos 2a\omega + \sin 2a\omega) e^{a^2-\omega^2} + \frac{1}{\sqrt{\pi}} \sum_{r=1}^{\infty} \sum_{m=0}^{M} \frac{(2m+r)!\sin\frac{\pi r}{2}}{2^{2m} r! m! \omega^{2m+r+1}} \cdot a^r. \tag{5}$$

The limits of applicability of the second method, discussed in ([6]) for the particular case of a homogeneous layer, remain the same in the present problem.

Table 1

Deviation function $F_c(k_0 l;\ n,\ a)$

n	a	$k_0 l =$ 2	$k_0 l =$ 4	$k_0 l =$ 8	$k_0 l =$ 12	$k_0 l =$ 16	$k_0 l =$ 24	$k_0 l =$ 32	$k_0 l =$ 40
$n =$ 1.5	0.01	2.7	7.1	16.0	21.8	25.7	30.6	34.4	36.6
$n =$ 1.5	0.1	2.5	6.7	15.1	19.9	22.7	25.4	26.8	27.7
$n =$ 1.5	0.4	2.1	5.5	11.6	14.5	16.2	17.5	18.4	19.0
$n =$ 1.5	0.6	1.8	5.0	9.9	12.6	14.1	15.3	16.0	16.3
$n =$ 1.5	1.0	1.5	4.2	9.0	10.7	11.8	13.0	13.6	13.9

Fig. 1

Figure 1: Fig. 1

n	a	$k_0l =$ 2	$k_0l =$ 4	$k_0l =$ 8	$k_0l =$ 12	$k_0l =$ 16	$k_0l =$ 24	$k_0l =$ 32	$k_0l =$ 40
$n =$ 1.5	2.0	1.2	3.5	8.3	9.8	10.2	10.6	10.9	11.0
$n =$ 2	0.01	4.7	11.6	25.1	32.9	37.3	43.7	47.5	49.2
$n =$ 2	0.1	4.4	10.8	23.1	30.0	33.6	37.4	39.3	41.0
$n =$ 2	0.4	4.0	9.2	17.5	21.7	23.5	25.2	26.1	27.2
$n =$ 2	0.6	3.5	8.2	15.6	19.3	21.3	23.0	24.0	25.1
$n =$ 2	1.0	2.8	7.0	14.0	17.9	19.8	21.6	22.7	23.8
$n =$ 2	2.0	2.1	6.3	12.5	16.3	18.1	20.0	21.1	22.0
$n =$ ∞	0.01	7.6	22.6	44.8	52.9	57.2	62.1	64.9	66.9
$n =$ ∞	0.1	7.0	20.2	42.1	47.6	50.1	53.0	54.8	56.2
$n =$ ∞	0.4	6.4	16.9	31.1	34.8	36.1	37.6	38.7	39.4
$n =$ ∞	0.6	6.0	15.0	27.1	30.6	32.0	33.4	34.3	35.0
$n =$ ∞	1.0	5.3	13.2	25.1	28.7	30.1	31.4	32.0	32.3
$n =$ ∞	2.0	4.6	11.2	22.1	26.0	27.5	28.9	29.3	29.4

Table 1 gives a series of values of the deviation function $F_c(k_0l;\ n, a)$, obtained with the aid of formula (4); the optical thickness of the layer is $k_0l = 0.0944\,x$. In the limiting cases $a = 0$ (Doppler contour) and $a = \infty$ (dispersion contour), the reabsorption function is calculated from the exact formulas derived in ([7]).

Fig. 1

II. In Fig. 1, the dashed and solid lines denote "growth curves" corresponding respectively to homogeneous ($n = 1$) and inhomogeneous ($n = 3$) emitters; in both cases $a = 0.1$. It is easy to see that the entire range of optical thicknesses covered by the generalized "growth curves" can be subdivided into five principal intervals:

1. The region of proportional transmission ($k_0 l < 0.5$), within which, when calculating the equivalent line width, the layer may be regarded as transparent.
2. The interval of a smoothly decreasing slope of the "growth curve" ($0.5 < k_0 l < 3.0$), corresponding to the onset of saturation of the line intensity at the center (homogeneous layer) or to the first stage of self-reversal (inhomogeneous layer). Deviations from a linear dependence appear here earlier and more noticeably the higher the degree of inhomogeneity, since in this case photons, on average, must overcome a greater optical thickness before leaving the luminous cloud.
3. The region of minimum slope ($3.0 < k_0 l < 15.0$); the magnitude of the latter is determined by the values of the Voigt parameter a and the degree of inhomogeneity n. The tangent of the angle of inclination will obviously be the smaller, the larger n is and the smaller a is. Indeed, the increment in the integral line intensity due to the wings of the contour is the weaker, the better this part of the contour is described by a Gaussian function, while the central depression of the line compensating it will turn out to be the more significant, the more inhomogeneous the layer is. Calculation shows that in the general case the tangent of the angle of inclination may also be negative.
4. The region where the "root law" is valid ($15 < k_0 l < 100$), i.e., the tangent of the angle of inclination is equal to 0.5. The increase in the equivalent width here, as is known, is practically entirely due to the wings of the contour, the intensity distribution in which obeys the simple dispersion formula:

$$\mu = \frac{0.0472\,\chi a e^{-a^2}}{[1 - \mathrm{erf}(a)]\sqrt{\pi}\,\omega^2}. \tag{6}$$

5. The region in which, owing to the asymmetry of the line contour and the deviation of the intensity distribution from the dispersion law, the tangent of the angle of inclination of the "growth curve" exceeds the value 0.5 ($k_0 l > 100$) ([7,10]).

It should be noted that the boundary values of the optical density of each of the intervals given above in parentheses were obtained as a result of averaging for "growth curves" with different a and n. As for the value $k_0 l = 100$, which bounds from below the region of least transparency of the layer, it is minimal and is realized under a combination of all the unfavorable factors causing deviations from formula (6).

III. A new and most interesting property of the generalized "growth curves" is the presence in some of them of a clearly anomalous segment, where an increase in optical density is accompanied not by an increase, but by a decrease in the equivalent width of the line. As already mentioned above,

Fig. 2

Figure 2: Fig. 2

this effect can appear at sufficiently small values of the Voigt parameter a, and the more readily, the higher the degree of plasma inhomogeneity.

Fig. 2

In Fig. 2 the shaded area indicates what part of the quadrant in the coordinates a–$1/n$ is occupied by the "anomaly zone." When a more general source model is used, the boundaries of this zone expand as the difference $[i - f(k = i, q)]$ decreases, if $i > 1$.

It is not difficult to estimate the experimental conditions that would favor the detection of an anomalous segment of the "growth curve." The source must have a significant temperature gradient in the direction in which the spectrum is observed, and the line under study must have a small Stark constant. In this case, to calculate the Voigt parameter a, one may write the formula

$$a = 5.5 \cdot 10^{-6} \lambda \sigma^2 p \bar{T}^{-1} \left(1 + m_1/m_2\right)^{1/2}, \tag{7}$$

where $\overline{T}$ is the mean plasma temperature; p is the pressure in torr; λ is the wavelength of the selected line in Å; σ^2 is the effective cross section for Lorentzian collision in Å^2; m_1 and m_2 are the atomic (molecular) weights for the components of the interacting pair. Thus relation (7) makes it possible to determine ways of possible minimization of the parameter a. We note that Bergstedt(11), solely on the basis of estimates of the quantities σ^2, succeeded in selecting a large group of spectral lines which, when excited in an arc discharge, had an almost undistorted Doppler contour.

IV. The generalized "growth curves" considered above can serve as a theoretical basis for justifying the course of spectroanalytical calibration graphs, and can also find various applications in other fields of spectroscopy and in astrophysics.

As an example, let us indicate one of the applications. Kostkowsky and Broida(12) recently proposed a new, very effective method for measuring population temperatures in hot gases by recording the minimum transparency of a luminous layer. According to the theory of this method, a specially constructed function of the minimum transparency must depend linearly on the energy of the lower level of a group of lines forming a rotational branch of an electronic transition in a diatomic molecule. The population temperature is then determined from the tangent of the angle of inclination of the graph. However, experiments carried out with excitation of OH molecules in flames revealed an unforeseen systematic bending of the graph, occurring in the region of the most intense lines. The authors(12) were unable to explain this phenomenon, which is very undesirable because it reduces the accuracy of the pyrometric procedure and also gives rise to doubts as to the existence of thermodynamic equilibrium.

Meanwhile, the indicated curvature of the graph can easily be understood if one takes into account the transverse temperature gradient of the flame[13]. Using formula (7) and Oldenberg' s data[14,15] on measurements of the widths of rotational lines in the spectrum of hydroxyl, it is not difficult to show that in the experiments of Kostkowsky and Broida the Voigt parameter is $a \cong 0.8$. Hence, from the observed curvature, using the "growth curves" calculated by us, one can ascribe to the flames with which the experiment[12] was performed a degree of inhomogeneity $n = 1.5 \div 1.8$. Recent direct measurements of the radial distribution of brightness and optical density in opaque flames[16] agree well with these estimates.

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Note: Figure translations are in progress. See original paper for figures.