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Abstract

Full Text

MECHANICS

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ON THE THEORY OF FOUR-LINK MECHANISMS WITH TWO PRISMATIC PAIRS

Consider the question of the line envelopes of the coupler curves of a mechanism (Fig. 1) formed by two revolute and two prismatic pairs with alternating arrangement. Let an arbitrary straight line $u-u$ be chosen in the plane of link 2. We shall find the instantaneous center of rotation P of link 2 and drop from the point P the perpendicular PM to the line $u-u$. The point M will belong to the curve that is the line envelope of the line $u-u$. Drop from the point A the perpendicular AF to the line $u-u$ and introduce the notation $AF = p$, $AE = p_1$, $AG = a$, and $EB = b$.

The equation for the family of straight lines $u-u$ will be

$$x\cos\theta + y\sin\theta = p. \tag{1}$$

or

$$x\cos(\theta_1+\gamma) + y\sin(\theta_1+\gamma) = p_1\cos\gamma. \tag{2}$$

Take the partial derivative with respect to the parameter θ_1 of expression (2):

$$y\cos(\theta_1+\gamma) - x\sin(\theta_1+\gamma) = \frac{\partial p_1}{\partial\theta_1}\cos\gamma. \tag{3}$$

The segment p_1 will be equal to

$$p_1 = \frac{a - b\sin\theta_1}{\cos\theta_1}. \tag{4}$$

The partial derivative $\partial p_1/\partial\theta_1 = \Psi$ will be equal to

$$\Psi = \frac{a\sin\theta_1 - b}{\cos^2\theta_1}. \tag{5}$$

From equations (2) and (3) we obtain the parametric equations of the desired line-envelope curve:

$$x = [p_1 \cos(\theta_1 + \gamma) - \Psi \sin(\theta_1 + \gamma)] \cos\gamma,$$
$$y = [p_1 \sin(\theta_1 + \gamma) + \Psi \cos(\theta_1 + \gamma)] \cos\gamma, \tag{6}$$

where the functions $p_1 = p_1(\theta_1)$ and $\Psi = \Psi(\theta_1)$ are determined by equations (4) and (5). If the angle $\gamma = 90^\circ$, then the line-envelope curve is transformed into the point A. For the angle $\gamma = 0$, the equations of the line envelope will have the form

$$x = p_1 \cos\theta_1 - \Psi \sin\theta_1,$$
$$y = p_1 \sin\theta_1 + \Psi \cos\theta_1. \tag{7}$$

The locus of the point F will be the podaria of the line-envelope curve considered, if the point A is chosen as the pole of the podaria. The polar equation of this podaria will be

$$\rho = \frac{a - b\sin(\theta - \gamma)}{\cos(\theta - \gamma)} \cos\gamma. \tag{8}$$

Passing to a rectangular coordinate system, we obtain

$$(x^2 + y^2)[a\cos\gamma - (x\cos\gamma + y\sin\gamma)]^2 = b^2\cos^2\gamma\,(y\cos\gamma - x\sin\gamma)^2. \tag{9}$$

Consequently, the podaire will be an algebraic curve of the 4th order.

If the angle $\gamma = 0$, then the podaire will be the locus of points E (Fig. 1), and the equation of the podaire will be

$$(a - x)^2(x^2 + y^2) = b^2y^2. \tag{10}$$

Equation (10) is the equation of the pan-kappa ([1]). For $b = a$ we obtain the equation of an algebraic curve of the 3rd order

$$y^2 = \frac{a^2x - (2a - x)x^2}{2a - x}. \tag{11}$$

Equation (11) is the equation of the right strophoid with point of self-intersection G. Indeed, passing to the coordinate system x_1Gy_1 (Fig. 1), we obtain the equation of the strophoid in the usual form

$$y_1^2 = \frac{a - x_1}{a + x_1}x_1^2. \tag{12}$$

From the mechanism shown in Fig. 1, the Lebeau mechanism ([2]), shown in Fig. 2, can be obtained. The parametric equations of the curve linearly enveloping

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

the straight line $u-u$ in the Lebeau mechanism can be obtained from equations (6), if in them one substitutes the values for p_1 and Ψ, respectively equal to $p_1 = b/\operatorname{tg}\theta_1$ and $\Psi = -b/\sin^2\theta_1$:

Fig. 1

$$\begin{aligned} x &= b\cos\gamma\left[\frac{\cos\theta_1\cos(\theta_1+\gamma)}{\sin\theta_1} + \frac{\sin(\theta_1+\gamma)}{\sin^2\theta_1}\right], \\ y &= b\cos\gamma\left[\frac{\cos\theta_1\sin(\theta_1+\gamma)}{\sin\theta_1} - \frac{\cos(\theta_1+\gamma)}{\sin^2\theta_1}\right]. \end{aligned} \tag{13}$$

Fig. 2

For angle $\gamma = 90^\circ$ the linearly enveloping curve transforms into the point A. For angle $\gamma = 0$, the equations of the linearly enveloping curve will be

$$\begin{aligned} x &= b\left(\frac{1+\cos^2\theta_1}{\sin\theta_1}\right), \\ y &= b\frac{\cos\theta_1}{\operatorname{tg}^2\theta_1}. \end{aligned} \tag{14}$$

The podaire of the curve described by equations (13), if point A is chosen as the pole, will be the locus of points F (Fig. 2).

The equation of this podaire in polar form will be

$$p = \frac{b\cos\gamma}{\operatorname{tg}(\theta-\gamma)}. \tag{15}$$

Passing to a rectangular coordinate system, we obtain the equation of the podaire in the form

$$(x^2+y^2)(y - x\operatorname{tg}\gamma)^2 = b^2\cos^2\gamma\,(x + y\operatorname{tg}\gamma)^2. \tag{16}$$

If the angle $\gamma = 0$, then the pole curve will be the locus of points E, and the equation of the pole curve will be the equation of the "kappa" curve [(1)]

Fig. 3

Figure 3: Fig. 3

$$b^2x^2 = y^2(x^2 + y^2). \tag{17}$$

From the mechanism shown in Fig. 1, by taking $b = 0$, one can obtain the mechanism shown in Fig. 3.

The parametric equations of the curve linearly enveloping the straight line $u - u$ can be obtained from equations (6), if in them one substitutes the values for ρ_1 and Ψ, respectively equal to $\rho_1 = a/\cos\theta_1$ and $\Psi = a\sin\theta_1/\cos^2\theta_1$:

$$\begin{aligned} x &= \frac{a}{\cos^2\theta_1}\cos(2\theta_1 + \gamma)\cos\gamma, \\ y &= \frac{a}{\cos^2\theta_1}\sin(2\theta_1 + \gamma)\cos\gamma. \end{aligned} \tag{18}$$

Equations (18) are the equations of a parabola. Indeed, let us drop from point A to the straight line $u - u$ the perpendicular AF and connect point F with point G. Then, as was shown earlier by us [(1)], the mechanism under consideration can be replaced by an equivalent mechanism (Fig. 3), in which link $2'$ enters into a prismatic pair with slider $1'$, rotating about the axis A, and into a revolute pair with slider $3'$, sliding along the axis FG. The straight line $u' - u'$, rigidly connected with link $2'$ and forming an angle of $90°$ with the direction AF, will always coincide with the straight line $u - u$. The parabola that is the curve linearly enveloping the straight line $u - u$ or $u' - u'$ will have, as its vertex, the point O—the foot of the perpendicular dropped from point A onto the direction FG—and, as its focus, point A. The pole curve of the parabola defined by equations (18) is the straight line FG.

Fig. 3

If we take the angle $\gamma = 0$, then the curve linearly enveloping the straight line $u - u$ will be a parabola with focus at point A and vertex at point G. The equation of this parabola will be

$$y^2 = 4a(a - x) \tag{19}$$

and its pole curve will be the straight line BG. If point A is chosen as the pole, then the pole curve of this parabola will be the straight line FG. The latter, in particular, proves the kinematic equivalence of the mechanism AF to the mechanism AB.

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Note: Figure translations are in progress. See original paper for figures.