Hydraulics

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Abstract

Full Text

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ON THE BASIC REGULARITIES OF THE MOTION OF HYDRO- AND AEROMIXTURES IN PIPES

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1. Depending on the disperse composition, rocks mixed with a liquid or gas form various two-component media. The most common of these are finely dispersed suspensions (particles 0.03–0.15 mm) and coarsely dispersed suspensions (particles 0.15–2.0 mm), as well as heterogeneous disperse (particles greater than 2.0 mm) and polydisperse systems.

In practical applications—first and foremost for hydraulic and pneumatic transport—two problems are of great importance: the calculation of hydraulic resistances and the determination of the maximum transporting capacity of the flow. An exact solution of these problems can be obtained only from the equations of the hydrodynamics of a heterogeneous liquid. Consideration [1–6] shows that a rigorous theoretical analysis is powerless here. In the present work an attempt is made to obtain an approximate solution of the indicated problems for individual types of hydro- and aeromixtures.

2. For a uniform and cross-sectionally steady turbulent flow of an incompressible liquid carrying sufficiently small solid particles, from the equations of the averaged motion [3], by transforming them to dimensionless form, one can obtain the criterion equation

$$\mathrm{Eu} = \varphi_1(\mathrm{Re}, \mathrm{Fr}, \mathrm{Fr}', s, \delta, a), \tag{1}$$

where Eu is the Euler number; Re is the Reynolds number; Fr and Fr' are the Froude numbers, respectively for the flow and for the particles; s is the averaged concentration; $\delta = d/D$; $a = (\rho_s - \rho_0)/\rho_0$; d and D are the sizes of the particles and of the flow (here and below generally accepted notation is used); note that if the resistance forces are expressed according to Stokes' law, then in equation (1) the Archimedes criterion $\mathrm{Ar} = gd^3a/\nu^2$ will enter instead of Fr' and a.

For individual conditions, approximate similarity of the phenomena of transport of solid particles can be obtained if the ratio of the forces acting on the particles is kept constant (for example, the ratio of the friction forces of the particles against the walls to the mass force of inertia for the prototype and model, etc.).

Fig. 1

Figure 1: Fig. 1

For ascending flows of hydro- and aeromixtures, under the condition that the gravitational forces of the particles affect the motion, equation (1) takes the form

$$\mathrm{Eu} = \varphi_2(\mathrm{Re}, \mathrm{Fr}^*, s, a), \tag{2}$$

where $\mathrm{Fr}^* = gD/(u - u_*)^2$; $s \equiv \mu$ for pneumatic transport; u_* is the velocity of constrained settling of the particles.

For horizontal flows of heterogeneous disperse systems, under the condition that the friction force of the particles against the lower wall of the flow predominantly affects the motion,

$$\mathrm{Eu} = \varphi_3(\mathrm{Re}, f, s, a), \tag{3}$$

where f is the generalized coefficient of particle friction.

For coarsely dispersed suspensions, under the condition that the gravitational forces of the particles and of the liquid predominantly affect the motion,

$$\mathrm{Eu} = \varphi_4(\mathrm{Re}, \mathrm{Fr}, \mathrm{Fr}', s, a). \tag{4}$$

For finely dispersed suspensions, under the condition of the predominant influence on the motion of the degree of saturation of the liquid with particles,

$$\mathrm{Eu} = \varphi_5(\mathrm{Re},\ s,\ a). \tag{5}$$

In pneumatic transport, when the influence of the gravitational forces of the flow is appreciable,

$$\mathrm{Eu} = \varphi_6(\mathrm{Re},\ \mathrm{Fr},\ \mu,\ a). \tag{6}$$

3. The use of criterion equations for processing experimental data should expediently be combined with approximate energy-balance equations. Following ([2,3]), let us consider uniform motion in a horizontal pipe of monodisperse media under the condition that the maximum transporting capacity of the flow is not fully realized.

Fig. 1

The desired equations for the plane problem can be obtained from the energy equation of the averaged motion for both components in the form

$$\sum_{i=1}^{3} u\left[-\sum_{k=1}^{3}\frac{\partial \bar{p}_{ik}}{\partial x_k}-\sum_{k=1}^{3}\frac{\partial \Pi_{ik}}{\partial x_k}-\sum_{k=1}^{3}\frac{\partial \Pi_{iks}}{\partial x_k}+\rho_0(1-\bar{s})X_i+\rho_s s X_{is}\right]=0$$

by transforming it on the basis of certain assumptions.

In the case of the motion of **finely dispersed suspensions**, owing to the smallness of the particles and their participation in the pulsational motion of the liquid, it is assumed that in the flow the excitation of the internal motions of mixing by the energy of the main motion is intensified by the presence of solid particles. Then the amount of energy contained in a unit element of the flow and equivalent to the work of the resistance forces caused by additional excitations of internal motions due to the presence of solid particles in the same volume is $c_0(\rho_s-\rho_0)\bar{u}_s\bar{s}(d\overline{u_s'v_s'}/dy)$. If one puts $\bar{u}_s=\bar{u}$, $\overline{u_s'v_s'}\approx\overline{u'v'}$, and

$$i=\left[(1-\bar{s})X_i/g+\rho_s s X_{is}/\rho_0 g\right],$$

then the energy equation takes the form

$$gui-\bar{u}\,\frac{d\overline{u'v'}}{dy}-c_0\frac{\rho_s-\rho_0}{\rho_0}\,\bar{u}s\,\frac{d\overline{u'v'}}{dy}=0. \tag{7}$$

Equation (7), at $y=D$ and under the condition that s is averaged over the entire depth, gives, in agreement with (5),

$$i=i_0(1+c_0as), \tag{8}$$

where i and i_0 are the specific pressure losses; c_0 is a constant taking into account the characteristics of flow mixing; according to measurements (by the author for anthracite, by Durand for sand, by Filatov for clay, etc.) in pipes $D=25$–250 mm, $c_0\approx 1$.

In the case of the motion of **coarsely dispersed suspensions**, the solid particles perform saltatory motions, mainly not associated with turbulent pulsations. Their influence on the motion appears in the form of a "retarding" of the flow; a new form of shear stress appears in the body of the flow and, most importantly, near the lower wall of the flow. The amount of energy equivalent to the work against the resistance forces caused by the relative motion of the particles along the vertical of the flow and by the new form of stress

friction near the lower wall, may be represented as: $c_1 g(\rho_s-\rho_0)u_*\times\bar{s}\sqrt{\delta_0}$; $\delta_0=1/\delta$, where c_1 is a constant allowing for the particular features of the motion.

From an energy-balance equation of the form (7), in accordance with (4), we have

$$i = i_0 + c_1 \frac{asu_*}{u}\sqrt{\delta_0}, \qquad c_1 = 0.3 - 0.45.$$

Measurement data in pipes $D = 150$-700 mm are given in Fig. 1 (1—the author; 2—Durand; $3, 4$, and 5—Klimentov; 6-11—Solei and Balyad).

Fig. 2

In the case of the transport of **nonuniform dispersed systems**, large particles create, in a certain region of the flow near the bottom, a mobile roughness (6). The "braking" of the flow caused by the roughness is the determining factor and is practically the same for particles of size 1.5-2 mm and larger. The amount of energy equivalent to the work against the resistance forces due, in addition to saturation of the flow with particles, to friction forces between the particles and the wall of the stream when they move with velocity $\bar{u}_s$, i.e. $fg(\rho_s - \rho_0)\bar{u}_s s$. From an energy-balance equation of the form (7), under the condition $\bar{u}_s \approx \bar{u}$ and in accordance with (3), we have: $i = i_0 + fas$, where $f = 0.7$-0.6 for freshly crushed rocks; $f = 0.6$-0.5 for soft rocks; $f = 0.3$-0.2 for crushed coal; $f = 0.45$-0.3 for rounded rocks; $f = 0.2$-0.1 for anthracite. Measurement data in pipes $D = 25$-400 mm are given in Fig. 2 ($1, 2, 5$, and 7—the author; $3, 4$, and 8—Trynis and Korzhaev; 6—Nevit; 9—Durand; 10—Worster and Denny).

Fig. 3

In the case of the motion of **polydisperse mixtures**, using the principle of superposition of resistances widely employed in hydrodynamics, the energy-balance equation may be written in the form:

$$g\bar{u}i\,dy - [1 + c_0 as_1]\bar{u}\,d(\overline{u'v'}) - c_1 a_* g s_2 \frac{u_*}{u}\sqrt{\delta_0}\,dy - fga_* s_3 \bar{u}\,dy = 0, \qquad (9)$$

where s_1, s_2, and s_3 are the concentrations of the finest, fine, and coarse particles, which together give s; $a_* = [\rho_s - (1 + as_1)]/(1 + as_1)$. From (9) we have

$$i = i_0(1 + c_0 as_1) + c_1 a_* \frac{s_2 u_*}{u}\sqrt{\delta_0} + fa_* s_3. \qquad (10)$$

Figure 3 presents calculation data according to (10) and experiment (agreement with experiment was obtained for coals and rocks according to data of the author and other investigators).

In the case of the motion of **aerosuspensions**, in accordance with (6), (8), and measurement data (of the author, Rose, Peter, Zegler, and others, for $D =$ 35–300 mm), at Froude numbers $\mathrm{Fr} > 1$,

$$i = i_0(1 + \alpha\mu), \qquad \alpha = 0.1\text{–}0.2;$$

Fig. 4

Figure 2: Fig. 4

at $\mathrm{Fr} < 1$,

$$\frac{i - i_0}{\mu i_0} = \frac{\alpha_1 agD}{u^2}, \qquad \alpha_1 \approx 0.08.$$

Fig. 4

In the case of the transport of solid particles by **ascending flows** at considerable velocities, the energy-balance equation is reduced to the form (8). For the phase of motion characterized by the influence of the gravitational forces of the particles, as follows from Fig. 4, according to measurement data for catalyst, sand, coal, etc., with grain size from 0.07 to 7 mm in pipes $D = 13$–300 mm (1—Rose, 2—Kinest, 3—Orone, 4—Harou, 5—Zegler, 6—Baru, 7—Fort, 8—Ritkhoepel) for hydromixtures and aerosuspensions, respectively, in agreement with (2),

$$i = i_0 \left[(1 + 6s_1), \frac{\sqrt{agD}}{(u - u_*)} \right],$$

$$i = i_0 \left[(1 + 0.8\mu), \frac{\sqrt{agD}}{(u - u_*)} \right],$$

where s_1 is the weight concentration of the hydromixture (μ is the same for the aerosuspension).

4. If the motion is characterized by limiting saturation with solid particles, then condition (2) is satisfied, according to which the ratio of the additional energy expenditures due to the presence of particles in the flow to the total energy expenditures is a constant quantity. On the basis of the above measurement data, it was obtained: for coarsely dispersed hydromixtures

$$u_{\mathrm{cr}} = c' \sqrt{D} \sqrt[3]{\frac{asu_*}{\sqrt{d_{\mathrm{av}}}}},$$

for nonuniform disperse systems and aerosuspensions, respectively,

$$u_{\mathrm{cr}} = c'' \sqrt{fagsD}, \qquad u_{\mathrm{cr}} = c''' \sqrt{\mu agD},$$

for polydisperse systems of hydromixtures

$$u_{\mathrm{cr}} = c' \sqrt{D} \sqrt[3]{\frac{a_* s_2 u_*}{7 \sqrt{d_{\mathrm{av}}}}} + c'' \sqrt{f_* g s_3 D},$$

where

$$c' = 7\text{–}9; \qquad c'' = 7.5\text{–}10; \qquad c''' = 0.3\text{–}0.45.$$

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Note: Figure translations are in progress. See original paper for figures.