Geophysics

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1961

SovietRxiv

Abstract

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Geophysics

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ISOLATION OF LONG WAVES IN THE HYDRODYNAMIC FORECASTING OF PRESSURE FIELDS IN THE MIDDLE AND UPPER TROPOSPHERE OVER THE NORTHERN HEMISPHERE OF THE EARTH

(Presented by Academician L. I. Sedov on 16 I 1961)

Starting from the work of E. N. Blinova ([2]), we shall give a solution to the problem of long-range forecasting for small disturbances of the heights of the 700- and 300-millibar isobaric surfaces, restricting ourselves to a linear approximation. To this end we linearize, with respect to the west–east transfer, the vorticity-change equation and the heat-inflow equation, using the same method as in ([2]). In the formulation adopted by us, the angular velocity of the zonal current may vary with height, but does not depend on latitude. The forecast is made on the basis of a two-level model of the atmosphere. The 700- and 300-mb surfaces are taken as the characteristic levels.

As boundary conditions we take the vertical velocity at the ground to be zero and the vertical mass flux at the upper boundary of the atmosphere to be zero. We describe the relation between the velocity field and the pressure field by means of the geostrophic relation $\psi = \frac{g}{l}z$, where ψ is the stream function, z is the height of the isobaric surface, and l is the Coriolis parameter. For simplicity of calculation we shall understand by l its mean value for the hemisphere, although it is not difficult to take into account its dependence on latitude.

The forecasting equations in this case reduce to the form (see ([3])).

$$\left(\frac{\partial}{\partial t} + \alpha_1\frac{\partial}{\partial\lambda}\right)\left[\Delta z_1 + \frac{1}{\Gamma}(z_1 - z_3)\right] + \left[2\omega + \frac{1}{\Gamma}(\alpha_1 - \alpha_3)\right]\frac{\partial z_1}{\partial\lambda} = 0,$$

$$\left(\frac{\partial}{\partial t} + \alpha_3\frac{\partial}{\partial\lambda}\right)\left[\Delta z_3 - \frac{1}{\Gamma}(z_1 - z_3)\right] + \left[2\omega - \frac{1}{\Gamma}(\alpha_1 - \alpha_3)\right]\frac{\partial z_3}{\partial\lambda} = 0, \qquad (1)$$

where z_1 and z_3 are the heights of the 300- and 700-millibar isobaric surfaces; α_1 and α_3 are the circulation indices on the same surfaces; ω is the angular velocity of the Earth' s rotation. Γ is the parameter introduced by E. N. Blinova ([2]), for which the mean value over the hemisphere is taken:

$$\Gamma = \left[\frac{(\gamma_a - \gamma)T_1}{4\omega^2\cos^2\theta}\right]_{\text{av}};$$

here T_1 is the height-mean value of the temperature; γ is the vertical temperature gradient, and γ_a is the adiabatic temperature gradient.

For values of the difference $\alpha_1 - \alpha_3$ greater than 0.037, system (1) admits, as shown in ([3]), unstable solutions, i.e. wave solutions whose amplitude grows exponentially with time.

In the stable case, the solution satisfying the initial data has the form

$$\begin{aligned} z_3 &= \sum_{m=1}^{\infty}\sum_{n=m}^{\infty}\left[M_n^m(t)\cos m\lambda + N_n^m(t)\sin m\lambda\right]P_n^m(\cos\theta), \\ z_1 &= \sum_{m=1}^{\infty}\sum_{n=m}^{\infty}\left[\overline{M}_n^m(t)\cos m\lambda + \overline{N}_n^m(t)\sin m\lambda\right]P_n^m(\cos\theta); \end{aligned} \tag{2}$$

$$\begin{aligned} M_n^m(t) &= C_{1n}^m\cos m\sigma_n^1 t + D_{1n}^m\sin m\sigma_n^1 t + C_{2n}^m\cos m\sigma_n^2 t + D_{2n}^m\sin m\sigma_n^2 t, \\ N_n^m(t) &= D_{1n}^m\cos m\sigma_n^1 t - C_{1n}^m\sin m\sigma_n^1 t + D_{2n}^m\cos m\sigma_n^2 t - C_{2n}^m\sin m\sigma_n^2 t. \end{aligned} \tag{3}$$

$\overline{M}_n^m$ and $\overline{N}_n^m$ are computed from formulas (3), replacing C_n^m and D_n^m by $\overline{C}_n^m$ and $\overline{D}_n^m$;

$$\begin{aligned} C_{1n}^m &= \beta_n^1 A_n^m + \beta_n^2\overline{A}_n^m, & \overline{C}_{1n}^m &= \beta_n^4 A_n^m + \beta_n^3\overline{A}_n^m, \\ D_{1n}^m &= \beta_n^1 B_n^m + \beta_n^2\overline{B}_n^m, & \overline{D}_{1n}^m &= \beta_n^4 B_n^m + \beta_n^3\overline{B}_n^m, \\ C_{2n}^m &= \beta_n^3 A_n^m - \beta_n^2\overline{A}_n^m, & \overline{C}_{2n}^m &= \beta_n^1\overline{A}_n^m - \beta_n^4 A_n^m, \\ D_{2n}^m &= \beta_n^3 B_n^m - \beta_n^2\overline{B}_n^m, & \overline{D}_{2n}^m &= \beta_n^1\overline{B}_n^m - \beta_n^4 B_n^m, \end{aligned} \tag{4}$$

$$\begin{aligned} \beta_n^1 &= -k_n^2\beta_n^2, & \beta_n^2 &= \frac{B - 2\Lambda V}{2\sqrt{B^2 - \Lambda^2(4-\Lambda^2)V^2}}, \\ \beta_n^3 &= k_n^1\beta_n^2, & \beta_n^4 &= \frac{B + 2\Lambda V}{2\sqrt{B^2 - \Lambda^2(4-\Lambda^2)V^2}}; \end{aligned} \tag{5}$$

$$k_n^{1,2} = 1 + \Lambda - \frac{B - 2V}{\sigma_n^{1,2} + \alpha_3},$$
$$\sigma_n^{1,2} = -\frac{\alpha_1 + \alpha_3}{2} + \frac{(1+\Lambda)B}{\Lambda(\Lambda+2)} \pm \frac{\sqrt{B^2 - \Lambda^2(4-\Lambda^2)V^2}}{\Lambda(\Lambda+2)}, \tag{6}$$
$$B = 2\omega\Gamma, \qquad \Lambda = n(n+1)\Gamma, \qquad V = \frac{1}{2}(\alpha_1 - \alpha_3);$$

$A_n^m, B_n^m, \overline{A}_n^m, \overline{B}_n^m$ are the coefficients of the expansion in spherical functions of the initial AT-700 and AT-300 fields, respectively.

The initial fields were taken by us from maps of the absolute topography of the Northern Hemisphere from $\theta = 5^\circ$ to $\theta = 90^\circ$ (θ is the complement of the latitude), with a step of 5° (18 latitudes in all). In longitude, the data were taken from $\lambda = 0^\circ$ to $\lambda = 350^\circ$, with a step of 10° (36 points in all). Thus, over the whole hemisphere the data are taken at 648 nodes of the degree grid. For a sufficiently accurate representation of the initial field at such a large number of points, many terms must be taken in series (2) ($m \leq 18$, $n \leq 36$). In all this gives, in series (2), 252 terms with different m and n. Such a large number of terms provides representation of the initial field with an accuracy of up to 1 dkm.

The indicated computational scheme was implemented on the "Ural" electronic computer. The entire computation is divided into three stages. At the first stage, the initial AT-700 and AT-300 fields are expanded into series in spherical functions, i.e., the coefficients $A_n^m, B_n^m, \overline{A}_n^m$ and $\overline{B}_n^m$ are computed. In addition, for the initial fields the zonal means are computed, and from them the circulation indices α_1 and α_3 are found.

At the second stage, by formulas (5) and (6), the values of the quantities $\beta_n^1, \beta_n^2, \beta_n^3, \beta_n^4, \sigma_n^1$ and σ_n^2, corresponding to the found value of the difference $\alpha_1 - \alpha_3$, are computed; then, by formulas (4) and (3), the functions M_n^m, N_n^m, $\overline{M}_n^m$, and $\overline{N}_n^m$ are found, i.e., a forecast of the expansion coefficients is obtained.

At the third stage, the predicted fields are reconstructed from the computed expansion coefficients. The computation is carried out according to formulas (2). Only the final results are printed. The intermediate quantities, i.e., the results of the first and second stages, are recorded and stored on magnetic tape.

To assess the success of the forecast, the quantities r and E were computed, where r is the correlation coefficient between the observed and predicted changes in the height of the isobaric surface, and E is the mean absolute error. In the assessment, not the whole hemisphere was taken, but the region bounded by the parallels $\theta = 20^\circ$ and $\theta = 50^\circ$ and by the meridians $\lambda = -20^\circ$ and $\lambda = 50^\circ$. This region, covering Western Europe and the European territory of the USSR, contains 56 points of our grid.

Whereas for the first day the correlation coefficient is rather high, on the following days it shows a substantial decrease (see Table 1). Such a marked decrease in the success of the forecast for the second and especially for the third day may be explained by the inaccurate allowance for long waves.

Table 1

	r	r	r	E	E	E
	1 day	2 days	3 days	1 day	2 days	3 days
Scheme 1	0.63	0.38	−0.02	4.1	8.2	13
Scheme 2	0.70	0.70	0.67	3.1	5.1	5.8

The point is that, strictly speaking, the quantities z_1 and z_3, entering the homogeneous linear equations (1), should be regarded not as the heights of the isobaric surfaces themselves, but rather as their nonstationary nonzonal parts. In 1943[1] and in 1946 E. N. Blinova proposed forecasting deviations of meteorological quantities from their climatic values, i.e., forecasting the "anomalies" of meteorological quantities. This is also applicable to our work.

Analysis of the actual material shows that in nature the long pressure waves are almost stationary: from day to day they undergo small phase displacements about some mean position. These waves may be approximately identified with climatic stationary waves. But then we can regard the "anomalies" of these large waves as zeros—they should not participate in our forecast. Bearing this in mind, we constructed forecasts assuming that the waves corresponding to the numbers $m = 1, 2$ and 3 are stationary. This procedure is analogous to that used by Wolff in compiling a nonlinear barotropic forecast[2].

Forecasts according to this scheme are verified considerably better. As an illustration we give an example of a forecast of an AT-700 chart. The situation on 7 January 1959 was taken as the initial one. Numerical estimates of the success for 1, 2, and 3 days are given in Table 1, where scheme 1 denotes the ordinary forecast, and scheme 2 denotes the forecast with stabilization of the long waves (the mean absolute error E is given in decameters). Figs. 1 and 2 illustrate the forecast for the third day according to scheme 2.

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Received
16 January 1961

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Note: Figure translations are in progress. See original paper for figures.

[1] E. N. Blinova, DAN, **39**, No. 7, 284 (1943).
[2] R. M. Wolff, *Monthly Weather Rev.*, **86**, No. 7, 245 (1958).